2011 offline

The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are vectors satisfying $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 0$ Then vector $\vec{d}$ is equal to

(a) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (b) $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(c) $\vec{c} + \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (d) $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

$$\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$$

$$\vec{b}\times\vec{c} - \vec{b}\times\vec{d} = 0$$

$$\vec{b}\times(\vec{c}-\vec{d}) = 0$$

$$\Rightarrow \vec{c}-\vec{d} = \lambda\vec{b} \quad \text{for } \lambda \in R$$

$$\vec{c} - \lambda\vec{b} = d \qquad \text{I}$$

$$\vec{a}\cdot\vec{d} = 0 \quad (\text{given})$$

$$\vec{a}\cdot(\vec{c}-\lambda\vec{b}) = 0$$

$$\vec{a}\cdot\vec{c} - \lambda\vec{a}\cdot\vec{b} = 0$$

$$\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}} = \lambda$$

I become $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b} = \vec{d}$

(a)

2011 off line

Let $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors which are pairwise non collinear, If $\vec{a}+3\vec{b}$ is co-llinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is

(a) a (b) c (c) 0 (d) a + c

$\vec{a}+3\vec{b}$ is collinear with $\vec{c}$

$\Rightarrow \vec{a}+3\vec{b} = \lambda\vec{c}$

$\vec{a}$ and $\vec{b}+2\vec{c}$ is collinear

$\vec{b}+2\vec{c} = \mu\vec{a}$

Now $\vec{a}+3\vec{b}+6\vec{c} = \lambda\vec{c}+6\vec{c} = \vec{c}(\lambda+6)$ I

$\vec{a}+3\vec{b}+6\vec{c} = \vec{a}+3(\vec{b}+2\vec{c})$

$= \vec{a}+3\mu\vec{a}$

$= \vec{a}(1+3\mu)$ II

$\Rightarrow$ From I and II

$\vec{c}(\lambda+6) = \vec{a}(1+3\mu)$

$\vec{a}$ and $\vec{c}$ are non collinear

$\Rightarrow \lambda+6 = 0 \quad 1+3\mu = 0$

$\Rightarrow \vec{a}+3\vec{b}+6\vec{c} = 0$ (c)

2011 offline

If the vector $p\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}$ $(p \neq q \neq r \neq 1)$ are c
then the value of $pqr-(p+q$

(a) 2 (b) 0 (c) -1 (d) -2

Vectors are coplanar

$$\Rightarrow \begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$p(qr-1)-1(r-1)+1(1-$

$pqr-p-r+1+1-q$

$pqr-p-q-r=-2$

$pqr-(p+q+r)=$

(d)

2019 offline

If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k}) \times \vec{b}$ then the possible value of

$(\vec{a}+\vec{b}) \cdot (-7\hat{i}+2\hat{j}+3\hat{k})$ is

(a) 0 (b) 3 (c) 4 (d) 8

$$\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k}) \times \vec{b}$$

$$\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) = -\vec{b} \times (2\hat{i}+3\hat{j}+4\hat{k})$$

$$\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) + \vec{b} \times (2\hat{i}+3\hat{j}+4\hat{k})$$

$$(\vec{a}+\vec{b}) \times (2\hat{i}+3\hat{j}+4\hat{k}) = 0$$

$\Rightarrow$ $\vec{a}+\vec{b}$ is parallel to $(2\hat{i}+3\hat{j}+4\hat{k})$

$$(\vec{a}+\vec{b}) = \lambda(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{I}$$

$$|\vec{a}+\vec{b}| = |\lambda(2\hat{i}+3\hat{j}+4\hat{k})|$$

$$\sqrt{29} = \pm\lambda\sqrt{2^2+3^2+4^2}$$

$[\because |\vec{a}+\vec{b}| = \sqrt{29}$ given$]$

$$\sqrt{29} = \pm\lambda\sqrt{29}$$

$$\lambda = \pm 1$$

I becomes $\vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$$(\vec{a}+\vec{b}) \cdot (-7\hat{i}+2\hat{j}+3\hat{k}) = \pm(2\hat{i}+3\hat{j}+4\hat{k}) \cdot (-7\hat{i}+2\hat{j}+3\hat{k})$$

$$= \pm(-14+6+12)$$

$$= \pm 4$$

(c)

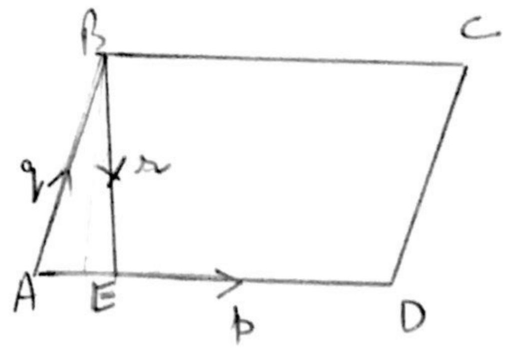
2012 off line

Let ABCD be a parallelogram such that $AB = q$ and $AD = p$ and $\angle BAD$ be an acute. If r is the vector that coincides with the altitude directed from the vertex B to the side AD, then ~~ra~~ r is given by

(a) $r = -q + \left(\frac{p \cdot q}{p \cdot p}\right)p$ (b) $r = q - \left(\frac{p \cdot q}{p \cdot p}\right)p$

(c) $r = -3q + 3\left(\frac{p \cdot q}{p \cdot p}\right)p$ (d) $r = 3q - 3\left(\frac{p \cdot q}{p \cdot p}\right)p$

$AE = \lambda p$ for $\alpha \in R$ I



BE is perpendicular on AD

$\Rightarrow BE \cdot AD = 0$

$r \cdot p = 0$ II

Now $BE = AE - AB$

$r = \lambda p - q$ $[\because AE = \lambda p]$

substitute in II

$(\lambda p - q) \cdot p = 0$

$\lambda p \cdot p - q \cdot p = 0$ $\lambda = \frac{q \cdot p}{p \cdot p}$

$\Rightarrow r = \left(\frac{q \cdot p}{p \cdot p}\right)p - q$ (a)

2012 off line

Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vector $\vec{c} = \vec{a} + 2\vec{b}$ and $\vec{d} = 5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

$|\vec{a}| = 1$ $|\vec{b}| = 1$

c and d are perpendicular to each other

$\Rightarrow \vec{c} \cdot \vec{d} = 0$

$(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$5|\vec{a}|^2 + 10\vec{b}.\vec{a} - 4\vec{a}.\vec{b} - 8|\vec{b}|^2 = 0$

$5 + 6\,\vec{a}.\vec{b} - 8 = 0$

$5 + 6\,|\vec{a}|\,|\vec{b}| \cos\theta - 8 = 0$

$6|\vec{a}||\vec{b}| \cos\theta = 3$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$ (b)

Scanned with CamScanner

2013 online

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector of the type $\vec{b} + \lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is

(a) $2\hat{i} + \hat{j} + 5\hat{k}$ (b) $2\hat{i} + 3\hat{j} - 3\hat{k}$

(c) $2\hat{i} - \hat{j} + 5\hat{k}$ (d) $2\hat{i} + 3\hat{j} - 5\hat{k}$

Projection of $\vec{b} + \lambda\vec{c}$ on $\vec{a}$

$$= \frac{(\vec{b} + \lambda\vec{c}) \cdot \vec{a}}{|\vec{a}|}$$

$$\Rightarrow \pm\sqrt{\frac{2}{3}} = \frac{(\vec{b} + \lambda\vec{c}) \cdot \vec{a}}{|\vec{a}|} \qquad \text{I}$$

$$a = 2\hat{i} - \hat{j} + \hat{k} \Rightarrow |a| = \sqrt{6}$$

$$b + \lambda c = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

$$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$$

$$(b+\lambda c) \cdot a = 2(1+\lambda) - (2+\lambda) - (1+2\lambda)$$

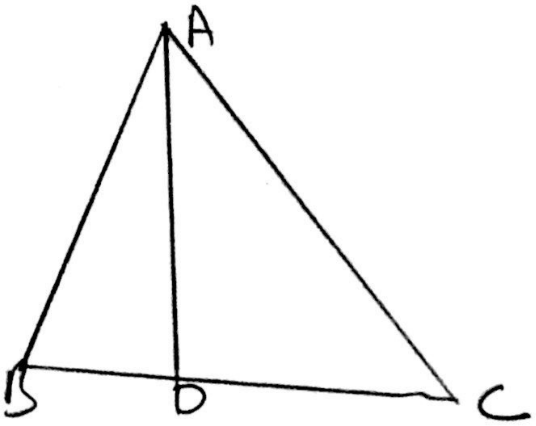
$$= \cancel{2} + \cancel{2\lambda} - \cancel{2} - \lambda - 1 - \cancel{2\lambda}$$

$$-\lambda - 1$$

2013 offline

If the vectors $AB = 3\hat{i} + 4\hat{k}$ and $AC = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(a) $\sqrt{72}$ (b) $\sqrt{33}$ (c) $\sqrt{45}$ (d) $\sqrt{18}$



$$AD = \frac{1}{2}(AB + AC)$$

$$= \frac{1}{2}\left((3\hat{i} + 4\hat{k}) + (5\hat{i} - 2\hat{j} + 4\hat{k})\right)$$

$$= \frac{1}{2}(8\hat{i} - 2\hat{j} + 8\hat{k})$$

$$= 4\hat{i} - \hat{j} + 4\hat{k}$$

$$|AD| = \sqrt{(4)^2 + (-1)^2 + (4)^2}$$

$$\sqrt{33}$$ (b)

2014 online

If $|\vec{c}|^2 = 60$ and $\vec{c} \times (\hat{i}+2\hat{j}+5\hat{k}) = 0$

Then a value of $\vec{c}\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(a) $4\sqrt{2}$ (b) 12 (c) 24 (d) $12\sqrt{2}$

$$\vec{c} \times (\hat{i}+2\hat{j}+5\hat{k}) = 0$$

$\Rightarrow (\hat{i}+2\hat{j}+5\hat{k})$ is $\parallel$ to $\vec{c}$

$\vec{c} = \lambda(\hat{i}+2\hat{j}+5\hat{k})$ for some $\lambda \in R$ (I)

$$|\vec{c}|^2 = \lambda^2(\hat{i}+2\hat{j}+5\hat{k})^2$$

$$60 = \lambda^2(1+4+25)$$

$$60 = \lambda^2(30)$$

$$\lambda^2 = 2$$

$$\vec{c}\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$$

$\lambda(\hat{i}+2\hat{j}+5\hat{k})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ (by I)

$$\lambda(-7+4+15)$$

$$\lambda(12)$$

$$12\sqrt{2}$$

(d)

2014 online

4. $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|2a-b$

Then $|2\vec{a}+\vec{b}|$ equals

(a) 17 (b) 7 (c) 5 (d) 1

$$|2\vec{a}+\vec{b}|^2 + |2\vec{a}-\vec{b}|^2$$

$$= 2\left(4|\vec{a}|^2 + |\vec{b}|^2\right)$$

$$= 2\left(4(2)^2 + (3)^2\right)$$

$$= 2(16+9)$$

$$= 50$$

$$\Rightarrow |2\vec{a}+\vec{b}|^2 + |2\vec{a}-\vec{b}|^2 = 50$$

$$|2\vec{a}+\vec{b}|^2 + (5)^2 = 50$$

$$|2\vec{a}+\vec{b}|^2 = 25$$

$$|2\vec{a}+\vec{b}| = 5$$

(c)

2014 online

If $\vec{x} = 3\hat{i} - 6\hat{j} - k$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ then the magnitude of the projection of $x \times y$ on z is

(a) 12 (b) 15 (c) 14 (d) 13

$$\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix}$$

$\vec{x} \times \vec{y} = 22\hat{i} + 8\hat{j} + 18\hat{k}$, $|\vec{z}| = \sqrt{9+16+144}$
$= 13$

Projection of $\vec{x} \times \vec{y}$ on $\vec{z}$

$$\frac{|(\vec{x} \times \vec{y}) \cdot \vec{z}|}{|\vec{z}|}$$

$$\frac{|(22\hat{i} + 8\hat{j} + 18\hat{k}) \cdot (3\hat{i} - 4\hat{j} - 12\hat{k})|}{13}$$

$$\frac{|66 - 32 - 216|}{13}$$

$$\frac{182}{13} = 14$$

(c)

2014 online

If $\vec{x}, \vec{y}$, and $\vec{z}$ are three unit vectors in three dimensional space, then the minimum value of

$$|\vec{x}+\vec{y}|^2 + |\vec{y}+\vec{z}|^2 + |\vec{z}+\vec{x}|^2 \text{ is}$$

(a) $\frac{3}{2}$ (b) 3 (c) $3\sqrt{3}$ (d) 6

$$|\vec{x}+\vec{y}|^2 + |\vec{y}+\vec{z}|^2 + |\vec{z}+\vec{x}|^2 - |\vec{x}+\vec{y}+\vec{z}|^2$$

$$= |\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}\cdot\vec{y} + |\vec{y}|^2 + |\vec{z}|^2 + 2\vec{y}\cdot\vec{z}$$
$$+ |\vec{z}|^2 + |\vec{x}|^2 + 2\vec{z}\cdot\vec{x} - (|\vec{x}|^2 + |\vec{y}|^2 + |\vec{z}|^2 +$$
$$2\vec{x}\cdot\vec{y} + 2\vec{y}\cdot\vec{z} + 2\vec{z}\cdot\vec{x})$$

$$= |\vec{x}|^2 + |\vec{y}|^2 + |\vec{z}|^2$$

$$= 3$$

$$\Rightarrow |\vec{x}+\vec{y}|^2 + |\vec{y}+\vec{z}|^2 + |\vec{z}+\vec{x}|^2 - |\vec{x}+\vec{y}+\vec{z}|^2 = 3$$

$$\Rightarrow |\vec{x}+\vec{y}|^2 + |\vec{y}+\vec{z}|^2 + |\vec{z}+\vec{x}|^2 = 3 + |\vec{x}+\vec{y}+\vec{z}|^2$$

So its minimum value is 3 which we get when $x+y+z=0$

(b)

2015

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between $\vec{b}$ and $\vec{K}$. Then value of $\sin\theta$ is

(a) $\frac{2\sqrt{2}}{3}$ (b) $-\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2\sqrt{3}}{3}$

$$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$(|\vec{a}||\vec{c}|\cos\phi)\vec{b} - (|\vec{b}||\vec{c}|\cos\theta)a = \frac{1}{3}|\vec{b}||\vec{c}|a$$

$$(|\vec{a}||\vec{c}|\cos\phi)\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|a + |\vec{b}||\vec{K}|\cos\theta$$

$$= |\vec{b}||\vec{c}|\vec{a}\left(\frac{1}{3} + \cos\theta\right)$$

$$= |\vec{b}||\vec{c}|\left(\frac{1}{3} + \cos\theta\right)\vec{a}$$

$\vec{a}$ and $\vec{b}$ are non collinear

$$\Rightarrow \cos\phi = 0, \quad \frac{1}{3} + \cos\theta = 0$$

$$\cos\theta = -\frac{1}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{1}{3}\right)^2 = 1$$

$$\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$$

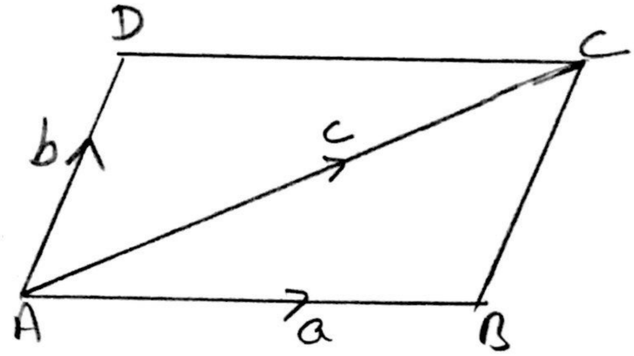
$$\sin\theta = \pm\frac{2\sqrt{2}}{3}$$

$\sin\theta = \frac{2\sqrt{2}}{3}$ (a)

2015 online

In a parallelogram ABCD, $|AB| = a$, $|AD| = b$ and $|AC| = c$ then DB . AB has the values

(a) $\frac{1}{2}(3a^2+b^2-c^2)$ (b) $\frac{1}{4}(a^2+b^2-c^2)$ (c) $\frac{1}{3}(b^2+c^2-a^2)$ (d) $\frac{1}{2}(a^2+b^2+c^2)$



$AC = AB + BC \Rightarrow c = a + b$

$DB = AB - AD \Rightarrow DB = a - b$

$DB \cdot AB = (a-b)\cdot a = a^2 - b\cdot a$

$c = |AC|$ (given)

$c = a + b$

$|c|^2 = |a+b|^2$

$c^2 = |a|^2 + |b|^2 + 2a\cdot b$

$c^2 - a^2 - b^2 = 2a\cdot b$

$\frac{1}{2}(c^2 - a^2 - b^2) = a\cdot b$

$AB = a^2 - a.b$

$= a^2 - \frac{1}{2}(c^2 - a^2 - b^2)$

$= 2a^2 - c^2 + a^2 + b^2$

$= \frac{1}{2}(3a^2 + b^2 - c^2)$

a

2015 online

Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}+\vec{b}| = \sqrt{3}$ If

$$\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a}\times\vec{b})$$

Then $2|\vec{c}|$ is equal to

(a) $\sqrt{55}$ (b) $\sqrt{51}$ (c) $\sqrt{43}$ (d) $\sqrt{37}$

$$|\vec{a}+\vec{b}| = \sqrt{3}$$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 3$$

$$1 + 1 + 2\vec{a}\cdot\vec{b} = 3$$

$$\vec{a}\cdot\vec{b} = \frac{1}{2}$$

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - \cancel{2}(\vec{a}\cdot\vec{b})^2$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a}\times\vec{b})$$

$$|\vec{c}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 9|\vec{a}\times\vec{b}|^2 + 4\vec{a}\cdot\vec{b} + 12\vec{b}\cdot(\vec{a}\times\vec{b}) + 6\vec{a}\cdot(\vec{a}\times\vec{b})$$

$$|\vec{c}|^2 = 1 + 4 + 9\times\frac{3}{4} + 4\left(\frac{1}{2}\right) + 0 + 0$$

[we know $\vec{b}\cdot(\vec{a}\times\vec{b}) = 0$
$\vec{a}\cdot(\vec{a}\times\vec{b}) = 0$]

$$= 1 + 4 + \frac{27}{4} + 2 = \frac{55}{4}$$

$$4|\vec{c}|^2 = 55 \Rightarrow 2|\vec{c}| = \sqrt{55}$$ (a)

2016 online

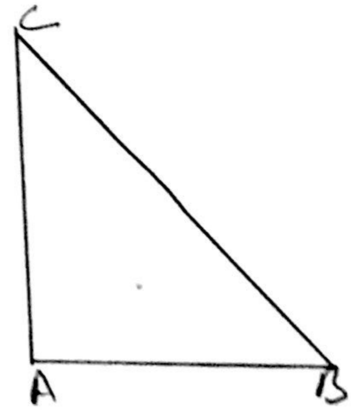
In a triangle ABC, right-angled at the vertex A. If the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}$, $-\hat{i}+3\hat{j}+p\hat{k}$, $5\hat{i}+q\hat{j}-4\hat{k}$ then the point (p, q) lies on the line

(a) making an obtuse angle with the positive direction of x-axis

(b) parallel to x-axis

(c) parallel to y-axis

(d) making an acute angle with the positive direction of x-axis



$\vec{AB} = -4\hat{i}+2\hat{j}+(p+1)\hat{k}$

$\vec{AC} = 2\hat{i}+(q-1)\hat{j}-3\hat{k}$

$\angle CAB = 90^\circ$

$\vec{AC}\cdot\vec{AB} = 0$

$(-4)(2)+2(q-1)+(p+1)(-3)=0$

$-8+2q-2-3p-3=0$

$-3p+2q-13=0$

$\Rightarrow (p,q)$ lies on $3x-2y+13=0$ which makes an acute angle with the x-axis

(d)

2016 online

Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are a, b, c and $\frac{1}{4}(a+b+c)$ respectively, then the position vector of the orthocentre of this triangle is

(a) $-\frac{1}{2}(a+b+c)$ (b) $a+b+c$

(c) $\frac{1}{2}(a+b+c)$ (d) 0

~~Cent~~ Circumcentre of triangle = P.

Let centroid = G orthocentre = H

we know centroid of a ~~triangle~~ triangle divide orthocentre and circumcentre at the ratio 2 : 1

$$H \overset{2}{\text{———}} G \overset{1}{\text{———}} P$$

Centroid $= \dfrac{a+b+c}{3}$

Position vector of P $= \frac{1}{4}(a+b+c)$

Let position vector of H = h.

$$\frac{a+b+c}{3} = \frac{2\left(\frac{1}{4}(a+b+c)\right)+h}{3}$$

$$(a+b+c) = \frac{1}{2}(a+b+c)+h$$

$$(a+b+c) - \frac{1}{2}(a+b+c) = h$$

$$\frac{1}{2}(a+b+c) = h$$

c

2016 offline

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

If $\vec{b}$ is not parallel to $\vec{c}$ then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$$

$$(\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

On comparing

$$\vec{a}.\vec{c} = \frac{\sqrt{3}}{2} \qquad \vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$$

we are to find angle between $\vec{a}$ & $\vec{b}$

$$\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$$

$$|\vec{a}|\,|\vec{b}|\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$\theta = \frac{5\pi}{6}$$

(d)