2011 off line

If $A = \sin^2 x + \cos^4 x$ then for all x

(a) $\frac{3}{4} \le A \le \frac{13}{16}$ (b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$ (d) $1 \le A \le 2$

$$A = \sin^2 x + \cos^4 x \le \sin^2 x + \cos^2 x =$$

$$\left[\because 0 \le \cos^2 x \le 1 \quad \cos^4 x \le \cos^2 x \right]$$

$$A = 1 - \cos^2 x + \cos^4 x$$

$$= \cos^4 x - \cos^2 x + 1$$

$$= y^2 - y + 1 \quad [\text{take } y =$$

$$= y^2 - y + \left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 + 1$$

$$= \left(y - \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$

$$= \left(\cos^2 x - \tfrac{1}{2}\right)^2 + \tfrac{3}{4} \ge \tfrac{3}{4}$$

$$\Rightarrow \tfrac{3}{4} \le A \le 1$$

(b)

2012 offline

In a triangle PQR if

$3\sin P + 4\cos Q = 6$ I and

$4\sin Q + 3\cos P = 1$ II

Then the angle R is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

Squaring and adding

$$(3\sin P + 4\cos Q)^2 + (4\sin Q + 3\cos P)^2 = 6^2 + 1^2$$

$$9\sin^2 P + 16\cos^2 Q + 24\sin P\cos Q + 16\sin^2 Q + 9\cos^2 P + 24\sin Q\cos P = 37$$

$$9(\sin^2 P + \cos^2 P) + 16(\cos^2 Q + \sin^2 Q) + 24\sin(P+Q) = 37$$

$$9 + 16 + 24\sin(P+Q) = 37$$

$$\sin(P+Q) = \frac{1}{2}$$

$$\sin(180 - R) = \frac{1}{2}$$

$$\sin R = \frac{1}{2}$$

$$R = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

R cannot be $\frac{5\pi}{6}$

$$\Rightarrow R = \frac{\pi}{6}$$

(d)

2012. offline

Q The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as

(a) $\sec A \operatorname{cosec} A + 1$ (b) $\tan A + \cot A$

(c) $\sec A + \operatorname{cosec} A$ (d) $\sin A \cos A + 1$

Sol

$$\frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$$

$$\frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A(1-\tan A)}$$

$$\frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

$$\frac{\cancel{(\tan A - 1)}(\tan^2 A + \tan A + 1)}{\tan A\,\cancel{(\tan A - 1)}}$$

$$\frac{\sec^2 A + \tan A}{\tan A}$$

$$\frac{\sec^2 A}{\tan A} + 1$$

$$\sec A \operatorname{cosec} A + 1$$

(a)

2013 online

The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is
(a) 3 (b) 4 (c) 5 (d) 6

$$\sin 2x - 2\cos x + 4\sin x - 4 = 0$$

$$2\sin x\cos x - 2\cos x + 4\sin x - 4 = 0$$

$$2\cos x(\sin x - 1) + 4(\sin x - 1) = 0$$

$$(\sin x - 1)(2\cos x + 4) = 0$$

$\sin x = 1$ or $2\cos x = -4$

$\cos x = -2$

$x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

which is not possible

no of solution = 3

a

2014 off line

Q Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and

Then $f_4(x) - f_6(x)$ equals

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

Sol $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x)$

$\frac{1}{6}(\sin^6 x + \cos$

$\frac{1}{4}(1 - 2\sin^2 x \cos^2 x) - \frac{}{6}$

$\frac{1}{4} - \cancel{\frac{2}{4_2}\sin^2 x \cos^2 x} - \frac{1}{6} + \frac{1}{2}$

$\frac{1}{4} - \frac{1}{6}$

$\frac{1}{12}.$

d

2014 online

Q The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$ is

(a) 6 (b) 4 (c) 3 (d) 1

Sol $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$2\sin^3\alpha - 2 - 7\sin^2\alpha + 7\sin\alpha = 0$

$2(\sin^3\alpha - 1) - 7\sin\alpha(\sin\alpha - 1) = 0$

$2(\sin\alpha - 1)(\sin^2\alpha + \sin\alpha + 1) - 7\sin\alpha(\sin\alpha - 1) = 0$

$(\sin\alpha - 1)\left[2(\sin^2\alpha + \sin\alpha + 1) - 7\sin\alpha\right] = 0$

$(\sin\alpha - 1)(2\sin^2\alpha + 2\sin\alpha + 2 - 7\sin\alpha) = 0$

$(\sin\alpha - 1)(2\sin^2\alpha - 5\sin\alpha + 2) = 0$

$(\sin\alpha - 1)(2\sin\alpha - 1)(\sin\alpha - 2) = 0$

$\sin\alpha - 1 = 0$	$2\sin\alpha - 1 = 0$	$\sin\alpha - 2 = 0$
$\sin\alpha = 1$	$\sin\alpha = \frac{1}{2}$	$\sin\alpha = 2$
$\alpha = \frac{\pi}{2}$	$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}$	which is not possible

There are three solutions

(c)

2014 advn

Q If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ $(p \neq q \neq 0)$ Then $\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ is equal to

(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$ (c) $\sqrt{pq}$ (d) pq

Sol $\operatorname{cosec}\theta = \frac{p+q}{p-q}$

$$\frac{\operatorname{cosec}\theta+1}{\operatorname{cosec}\theta-1} = \frac{p+\cancel{q}+p-\cancel{q}}{\cancel{p}+q-\cancel{p}+q}$$

$$\frac{\operatorname{cosec}\theta+1}{\operatorname{cosec}\theta-1} = \frac{p}{q}$$

$$\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2}-1}{1+\cot\frac{\pi}{4}\cot\frac{\theta}{2}}$$

$$= \frac{\cot\frac{\theta}{2}-1}{1+\cot\frac{\theta}{2}}$$

$$= \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$= \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$ (Rationalise)

$$= \frac{\cos\theta}{1+\sin\theta}$$

$$\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{\cos\theta}{1+\sin\theta}$$

$$\cot^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{\cos^2\theta}{(1+\sin\theta)^2}$$

$$= \frac{1-\sin^2\theta}{(1+\sin\theta)^2}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1}$$

$$= \frac{q}{p}$$

$$\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \sqrt{\frac{q}{p}}$$

2014 online

Q If $2\cos\theta + \sin\theta = 1$ $(\theta \neq \frac{\pi}{2})$ then $7\cos\theta + 6\sin\theta$ is equal to

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{11}{2}$ (d) $\frac{46}{5}$

Sol. $2\cos\theta + \sin\theta = 1$

$$2\cos\theta = 1 - \sin\theta$$

$$4\cos^2\theta = (1-\sin\theta)^2$$

$$4(1-\sin^2\theta) = (1-\sin\theta)^2$$

$$4(1+\sin\theta) = 1-\sin\theta$$

$$5\sin\theta = -3$$

$$\sin\theta = -\frac{3}{5}$$

$$2\cos\theta = 1-\sin\theta$$

$$2\cos\theta = 1+\frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$7\cos\theta + 6\sin\theta$$

$$7\left(\frac{4}{5}\right) + 6\left(-\frac{3}{5}\right)$$

$$\frac{28}{5} - \frac{18}{5} = \frac{10}{5} = 2$$

(b)

2014 online

Q If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$

and A and B are resp. the max and min values of $f(\theta)$. Then (A, B) is equal to

(a) $(3, -1)$ (b) $(4, 2-\sqrt{2})$ (c) $(2+\sqrt{2}, 2-\sqrt{2})$ (d) $(2+\sqrt{2}, -1)$

Sol

$$\begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$$

After solving it

$$2 + \sin 2\theta + \cos^2\theta - \sin^2\theta$$

$$2 + \sin 2\theta + \cos 2\theta$$

$$2 + \sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right)$$

if $\sin\left(2\theta + \frac{\pi}{4}\right) = 1$

Then Max value $A = 2 + \sqrt{2}$

if $\sin\left(2\theta + \frac{\pi}{4}\right) = -1$

Min value $B = 2 - \sqrt{2}$

$(2+\sqrt{2}, 2-\sqrt{2})$. (c)

2015 online

Q 21. $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β then $\sin 2\theta$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{7}{5}$ (d) $\frac{8}{5}$

Sol $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{3}{2}$ I

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}$ II

Divide I & II

$$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{3}.$$

$$\theta = \frac{\alpha+\beta}{2} \quad \text{(given.)}$$

$$\text{so } \tan\theta = \frac{1}{3}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^2}$$

$$= \frac{\frac{2}{3}}{1+\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{10}{9}}$$

$$\frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$$

(a)

2015 online

Q In a triangle $\frac{a}{b} = 2 + \sqrt{3}$ and $\angle C = 60^\circ$

Then the ordered pair

$(\angle A, \angle B)$ is equal to

(a) $(15^\circ, 105^\circ)$ (b) $(105^\circ, 15^\circ)$

(c) $(45^\circ, 75^\circ)$ (d) $(75^\circ, 45^\circ)$

Sol $\angle C = 60$

$\angle A + \angle B + \angle C = 180$

$\angle A + \angle B = 120^\circ$

$$2 + \sqrt{3} = \frac{a}{b} = \frac{\sin A}{\sin B}$$

$$2 + \sqrt{3} = \frac{\sin(120 - B)}{\sin B}$$

$$(2 + \sqrt{3})\sin B = \sin 120 \cos B - \cos 120 \sin B$$

$$(2 + \sqrt{3})\sin B \mp \cos 120 \sin B = \sin 120 \cos B$$

$$(2 + \sqrt{3})\sin B - \frac{1}{2}\sin B = \sin(180 - 60)\cos B$$

$$\left(2 + \sqrt{3} - \frac{1}{2}\right)\sin B = \cos 60 \cos B$$

$$\left(\frac{3 + 2\sqrt{3}}{2}\right)\sin B = \frac{\sqrt{3}}{2}\cos B$$

$$\cot B = \frac{\sqrt{3}}{3 + 2\sqrt{3}}$$

After rationalising we will get-

$\tan B = 2-\sqrt{3}$

$\Rightarrow \angle B = 15^\circ$

$\angle C = 60^\circ$

$\angle A = 105^\circ$

$(\angle A, \angle B)$ $(105^\circ, 15^\circ)$

(b)

2016 online

Q. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$
$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$
be two sets Then

(a) $P \subset Q$ and $Q - P \neq \phi$
(b) $Q \not\subset P$ (c) $P = Q$ (d) P

SOL. For set P

$$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$$

$$\sin\theta = \cos\theta + \sqrt{2}\cos\theta$$

$$\sin\theta = \cos\theta(1+\sqrt{2})$$

$$\frac{\sin\theta}{\cos\theta} = 1+\sqrt{2} \Rightarrow \tan\theta = \sqrt{}$$

For set Q

$$\sin\theta + \cos\theta = \sqrt{2}\sin\theta$$

$$\cos\theta = \sqrt{2}\sin\theta - \sin\theta$$

$$\cos\theta = \cancel{\sqrt{}}\sin\theta(\sqrt{2}-1)$$

$$\frac{1}{\sqrt{2}-1} = \tan\theta$$

$$\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \tan\theta$$

$$\sqrt{2}+1 = \tan\theta \quad \text{II}$$

From I & II

$$P = Q$$

2016 online

Q 21. m and M are the minimum and the maximum values of

$4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in R$

then M-m is equal to

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$ (c) $\frac{7}{4}$ (d) $\frac{1}{4}$

Sol

$$4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}\times 2 \times 2\cos^4 x.$$

$$4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}(2\cos^2 x)^2$$

$$4 + \frac{1}{2}(1 - \cos^2 2x) - \frac{1}{2}(1 + \cos 2x)^2$$

$$4 + \frac{1}{2}(1 - \cos 2x)(1 + \cos 2x) - \frac{1}{2}(1 + \cos 2x)^2$$

$$4 + \frac{1}{2}(1 + \cos 2x)\left[\cancel{1} - \cos 2x - \cancel{1} - \cos 2x\right]$$

$$4 - \frac{1}{\cancel{2}}(1 + \cos 2x)(\cancel{2}\cos 2x)$$

$$4 - (\cos 2x + \cos^2 2x)$$

$$4 - \cos 2x - \cos^2 2x.$$

$$4 - (\cos 2x + \cos^2 2x)$$

$$4 - (y^2 + y) \quad \left[\text{Let } \cos 2x = y\right.$$

$$4 - \left(y^2 + y + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)$$

$$4 - \left(\left(y + \frac{1}{2}\right)^2 - \frac{1}{4}\right)$$

$$4 - \left(\left(\cos 2x + \frac{1}{2}\right)^2 - \frac{1}{4}\right)$$

$$\frac{17}{4} - \left(\cos 2x + \frac{1}{2}\right)^2$$

$$m = \frac{17}{4} - \frac{9}{4} = 2. \qquad \left[\text{Take } \cos 2x = 1\right]$$

$$M = \frac{17}{4}$$

$$M - m = \frac{17}{4} - 2 = \frac{9}{4}$$

(a)

2016 offline

Q If $0 \le x < 2\pi$ then the number of real values of x which satisfy the equal
$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is

(a) 3 (b) 5 (c) 7 (d) 9

Sol $\cos x + \cos 3x + \cos 2x + \cos 4x = 0$

$2\cos 2x \cos x + 2\cos 3x \cos x = 0$

$2\cos x(\cos 2x + \cos 3x) = 0$

$2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$

$\cos x = 0 \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos\frac{x}{2} = 0 \qquad \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos\frac{5x}{2} = 0 \qquad \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

$x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

So $x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5},$

No of real values of x is 7

(c)

2016 online

Q The number of distinct real roots of the equation

$$\Delta \begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is

(a) 1 (b) 4 (c) 2 (d) 3

Sol Using $C_1 \rightarrow C_1 + C_2 + C_3$ we get-

$$\Delta = (2\sin x + \cos x) \begin{vmatrix} 1 & \sin x & \sin x \\ 1 & \cos x & \sin x \\ 1 & \sin x & \cos x \end{vmatrix}$$

After solving it we get.

$$(2\sin x + \cos x)(\cos x - \sin x)^2 = 0$$

So either $2\sin x + \cos x = 0$ or $\cos x - \sin x =$

$2\sin x = -\cos x$. $\cos x = \sin x$

$\tan x = -\frac{1}{2}$ $\tan x = 1$

$\tan x = 1$

As $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

$-1 \le \tan x \le 1$

There are two values of x

(c)

Q. 24- $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

Then the value of $\cos 4x$

(a) $-\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{9}$ (d) $-\frac{7}{9}$

$$5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$$

$$5\left(\frac{\sin^2 x}{\cos^2 x} - \cos^2 x\right) = 2\cos 2x + 9$$

$$5\left(\frac{\frac{1-\cos 2x}{2}}{\frac{1+\cos 2x}{2}} - \frac{1+\cos 2x}{2}\right) = 2\cos 2x + 9$$

$$5\left(\frac{1-\cos 2x}{1+\cos 2x} - \frac{1+\cos 2x}{2}\right) = 2\cos 2x + 9$$

Let $\cos 2x = y$

$$5\left(\frac{1-y}{1+y} - \frac{1+y}{2}\right) = 2y + 9$$

$$5\left(\frac{2-2y-(1+y)^2}{2(1+y)}\right) = 2y + 9$$

After solving we get

$$13 + 9y^2 + 42y = 0$$

$$(3y+1)(3y+13) = 0$$

$$y = -\frac{1}{3} \quad y = -\frac{13}{3}$$

$$\cos 2x = -\frac{1}{3} \quad \cos 2x = -\frac{13}{3}$$

$\cos 2x \neq -\frac{13}{3}$

so $\cos 2x = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1$

$= 2\left(-\frac{1}{3}\right)^2 - 1$

$= 2 \times \frac{1}{9} - 1$

$= \frac{2}{9} - 1$

$= -\frac{7}{9}$ (d)