2011 offline

Q. $A(2,-3)$, $B(-2,1)$ are two vertices of a triangle and the third vertex moves on the line $2x+3y=9$ then the locus of the centroid of the triangle is

(a) $x-y=1$ (b) $2x+3y=1$ (c) $2x+3y=3$

(d) $2x-3y=1$

$A(2,-3)$, $B(-2,1)$ $C(x,y)$ are the vertices of triangle ABC

Let $G(h,k)$ be the centroid of ΔABC

$$\frac{\cancel{2}-\cancel{2}+x}{3}=h.$$

$$x=3h.$$

$$\frac{-3+1+y}{3}=k$$

$$y=3k+2.$$

Co-ordinates of C are $(3h, 3k+2)$

C lies on the ~~line~~ line $2x+3y=9$

$$2(3h)+3(3k+2)=9$$

$$6h+9k+6=9$$

$$6h+9k=3$$

$$2h+3k=1$$

Locus of centroid $2x+3y=1$

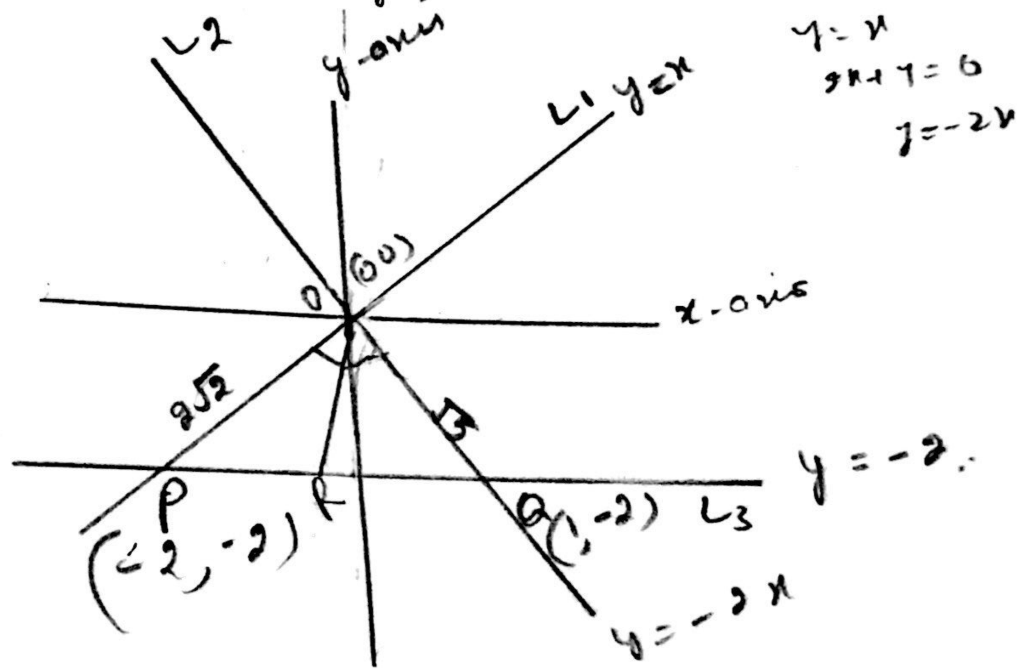
(b)

2011 offline

The line $L_1: y - x = 0$ and $L_2: 2x + y = 0$ intersect the line $L_3: y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersect L_3 at R.

Statement 1: The ratio of PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement 2: In any triangle, bisector of an angle divides the triangle into two similar triangles.



Intersecting point of L_1 and L_3 is $P(-2, -2)$

Intersecting point of L_2 and L_3 is $Q(1, -2)$

Length of $OP = 2\sqrt{2}$

Length of $OQ = \sqrt{5}$

2012

Q. 27. The line $2x + y = k$ passes through the point which divides the segment joining the points $(1,1)$ and $(2,4)$ in the ratio $3:2$, then k equals

(a) 6 (b) 11/5 (c) 29/5 (d) 5

$$(1,1) \; \underset{P(x,y)}{\overset{3 \;:\; 2}{\rule{3cm}{0.4pt}}} \; (2,4)$$

$$x = \frac{3 \times 2 + 2 \times 1}{3 + 2} = \frac{6 + 2}{5} = \frac{8}{5}$$

$$y = \frac{3 \times 4 + 2 \times 1}{5} = \frac{12 + 2}{5} = \frac{14}{5}$$

$\left(\frac{8}{5}, \frac{14}{5}\right)$ lies on the line $2x +$

$$2\left(\frac{8}{5}\right) + \frac{14}{5} = k$$

$$\frac{16}{5} + \frac{14}{5} = k$$

$$6 = k$$

(a)

In $\Delta\, OPQ$, OR bisect the acute angle $\angle POQ$

$$\Rightarrow \frac{OP}{OQ} = \frac{PR}{RQ}$$

$$\frac{2\sqrt{2}}{\sqrt{5}} = \frac{PR}{RQ}$$

$PR : RQ \quad 2\sqrt{2} : \sqrt{5}$

statement - 2 is not correct

2012 offline

Q. A line is drawn through the point (1,2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(a) -2 (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) -4.

Sol. Let equation of line through (1,2)

$$y-2 = m(x-1)$$

It meet x-axis when $y=0$

$$0-2 = mx - m$$

$$\frac{m-2}{m} = x. \quad \text{at } \left(\frac{m-2}{m}, 0\right)$$

It meet y-axis then $x = 0$

$$y-2 = m(0-1)$$

$$y = 2-m \quad \text{at } (0, 2-m)$$

$$\text{Area of OPQ} = \frac{1}{2}\left|\frac{m-2}{m}\right| |2-m|$$

$$= \frac{1}{2}\frac{(m-2)^2}{|m|}$$

$$= \frac{1}{2}\frac{m^2+4-4m}{|m|}$$

$$= \frac{1}{2}\left[|m| + \frac{4}{|m|} - \frac{4m}{|m|}\right]$$

$$\frac{1}{2}\left[|m| + \frac{4}{|m|} - 4 + 4 - \frac{4m}{|m|}\right]$$

$$\frac{1}{2}\left[\left(\sqrt{|m|} - \frac{2}{\sqrt{|m|}}\right)^2 + 4\left(1 - \frac{m}{|m|}\right)\right]$$

A is least if $|m| = 2$

i.e if $m = \pm 2$

$m = 2$ is not possible

[$\because$ then line passes through origin]

So $m = -2$

(a)

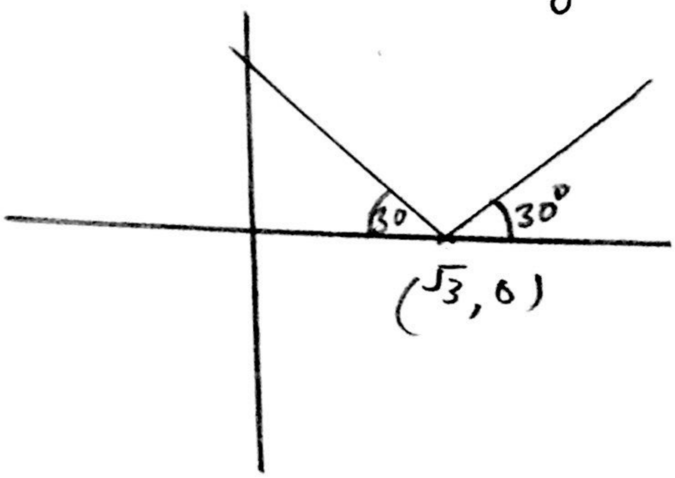
2013 offline

A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching the x-axis, an equation of the reflected ray is

(a) $\sqrt{3}y = x - \sqrt{3}$ (b) $y = \sqrt{3}x - \sqrt{3}$

(c) $\sqrt{3}y = x - 1$ (d) $y = x + \sqrt{3}$

The ray of light meet x-axis at $(\sqrt{3}, 0)$ [$\because$ Put $y = 0$ you will get $x = \sqrt{3}$]



Reflect ray will make an angle 30° with the x-axis

Now point is $(\sqrt{3}, 0)$

$$\text{slope} = \tan 30 = \frac{1}{\sqrt{3}}$$

Equation of reflected ray is

$$y - 0 = \frac{1}{\sqrt{3}}(x - \sqrt{3})$$

$$\sqrt{3}y = x - \sqrt{3}$$ (a)

2013 online

Equation of line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is

(a) $y - 3 = 0$ (b) $y + 3 = 0$ (c) $3y + 1 = 0$

(d) $3y - 1 = 0$

Find point of intersection of parabola and ellipse

$x^2 = 8y$ substitute this value of x^2 in $\frac{x^2}{3} + y^2 = 1$

$$\frac{8y}{3} + y^2 = 1$$

$$8y + 3y^2 = 3$$

$$3y^2 + 8y - 3 = 0$$

$$3y^2 + 9y - y - 3 = 0$$

$$3y(y+3) - (y+3) = 0$$

$$(y+3)(3y-1) = 0 \Rightarrow y = -3, \frac{1}{3}$$

$y \neq -3$ [$\because$ then $x^2 = 8(-3) = -24$ which is not possible]

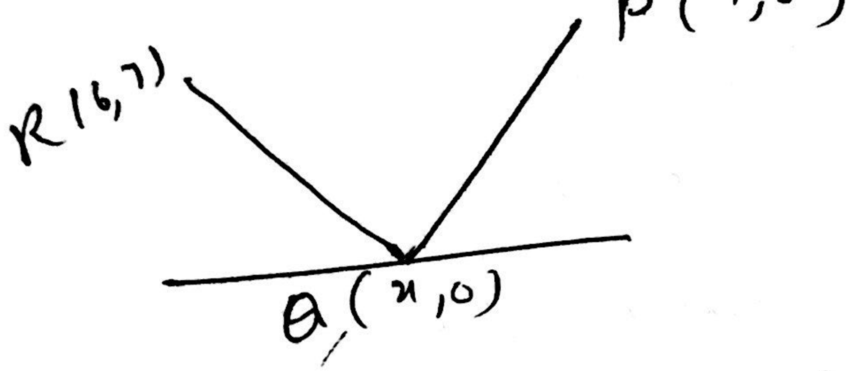
so $y = \frac{1}{3}$ i.e. satisfies $3y - 1 = 0$

(d)

2013 online

Q. A light ray emerging from the point source placed at P(1,3) is reflected at a point Q in the axis of x. If reflected ray passes through the point R(6,7) Then the abscissa of Q is

(a) 1 (b) 3 (c) $\frac{7}{2}$ (d) 5/2



slope of PQ = − slope of RQ

$$\frac{3}{1-x} = -\frac{7-0}{6-x}$$

$$\frac{3}{1-x} = \frac{7}{6-x}$$

$$18-3x = 7-7x$$

$$4x = -11$$

$$x = \frac{-11}{4}$$

2013 online

Q. If three lines $x - 3y = p$, $ax + 2y$
and $ax + y = r$ form right angled $\triangle$
Then (a) $a^2 - 9a + 18 = 0$ (b) $a^2 - 6a +$
(c) $a^2 - 6a - 18 = 0$ (d) $a^2 - 9a + 12$

Sol $x - 3y = p$ $ax + 2y = q$

$3y = x - p$ $2y = -ax + q$

$y = \frac{1}{3}x - p$ $y = -\frac{a}{2}x + q$

slope $= \frac{1}{3}$ slope $= -\frac{a}{2}$

Two lines are perpendicular.

if $\left(\frac{1}{3}\right)\left(-\frac{a}{2}\right) = -1$ $-a \times \frac{1}{3} =$

$-a = -6$ $a = 3$

$a = 6$

a satisfies the equation

$a^2 - 9a + 18 = 0$

a

2013 online

If x-intercept of some line L is double as that of the line $3x+4y=12$ and the y-intercept of L is half as that of the same line, then the slope of L is

(a) -3 (b) $-\frac{3}{8}$ (c) $-\frac{3}{2}$ (d) $-\frac{3}{16}$

$$3x+4y=12$$

$$\frac{x}{4}+\frac{y}{3}=1$$

Equation of L $\frac{x}{8}+\frac{y}{3/2}=1$

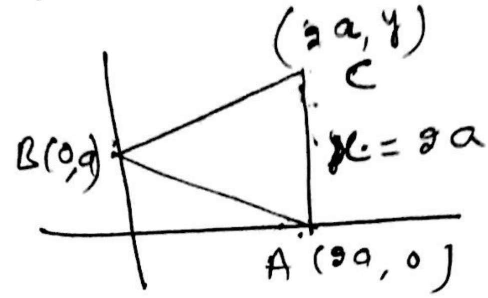
slope of $L = -\frac{3}{2}\times\frac{1}{8} = -\frac{3}{16}$

(d)

2012 online

Q. 4. The extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$ Then the area of the triangle in square units

(a) $\frac{5}{4}a^2$ (b) $\frac{5}{2}a^2$ (c) $\frac{25}{4}a^2$ (d) $5a^2$



$C \quad (2a, y)$

$BC = AC$

$\therefore BC^2 = AC^2$

$(0-2a)^2 + (a-y)^2 = (2a-2a)^2 + (0-y)^2$

$4a^2 + a^2 + \cancel{y^2} - 2ay = \cancel{y^2}$

$5a^2 - 2ay = 0$

$5a^2 = 2ay$

$y = \frac{5a}{2}$

Area of Δ $\quad \frac{1}{2} \times \text{Base} \times \text{height}$

$\frac{1}{\cancel{2}} \times y \times \cancel{2}a$

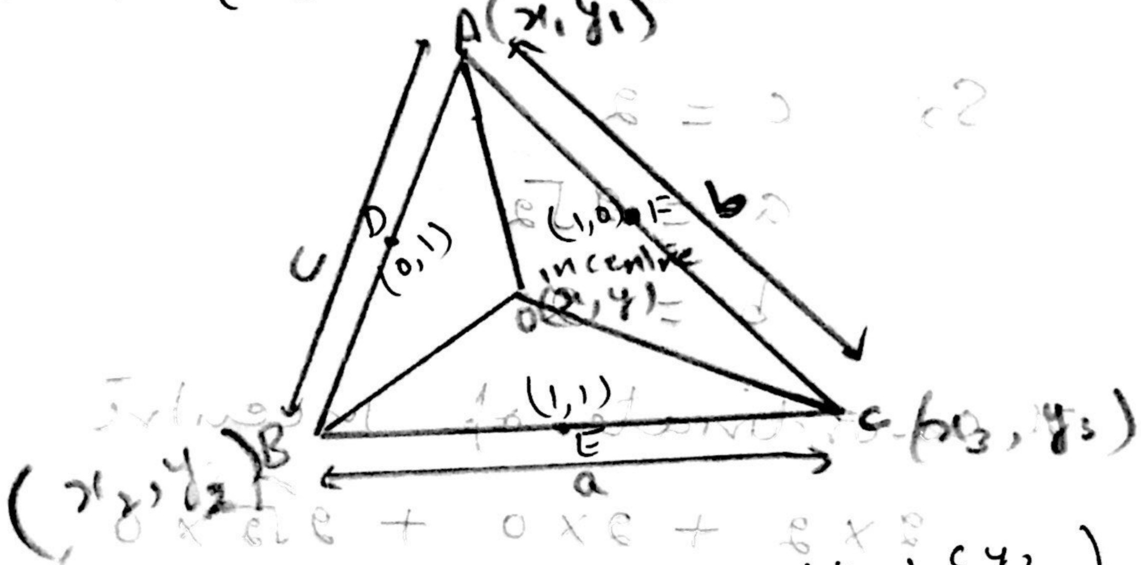
$= ya$

$= \frac{5a}{2} \times a = \frac{5}{2}a^2$

(b)

2013 offline

The x - co-ordinates of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is
(a) $2-\sqrt{2}$ (b) $1+\sqrt{2}$ (c) $1-\sqrt{2}$ (d) $2+\sqrt{2}$



$$\left(\frac{a x_1 + b x_2 + c x_3}{a+b+c}, \frac{a y_1 + b y_2 + c y_3}{a+b+c} \right)$$

are co-ordinates of incentre

i.e $x = \frac{a x_1 + b x_2 + c x_3}{a+b+c}$ and $y = \frac{a y_1 + b y_2 + c y_3}{a+b+c}$

Co-ordinates of A $(0+\cancel{1}-\cancel{1}, \cancel{1}+0-\cancel{1})$

A (0, 0)

Co-ordinates of B $(0+\cancel{1}-\cancel{1}, 1+1-0)$

B (0, 2)

Co-ordinates of C $(1+1-0, \cancel{1}+0-\cancel{1})$

(2, 0)

$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$

$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$

So $c = 2$

$a = 2\sqrt{2}$

$b = 2$

x co-ordinate of incentre

$$\frac{2\times 2 + 2\times 0 + 2\sqrt{2}\times 0}{2+2+2\sqrt{2}}$$

$$\frac{4}{4+2\sqrt{2}} \times \frac{4-2\sqrt{2}}{4-2\sqrt{2}}$$

$$\frac{4(4-2\sqrt{2})}{8}$$

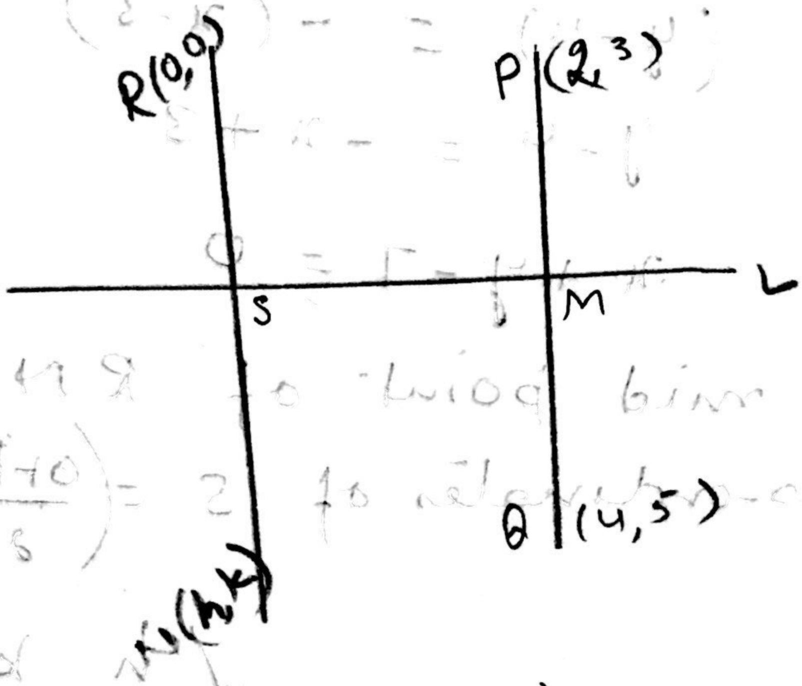
$$\frac{\cancel{4\times 2}(2-\sqrt{2})}{\cancel{8}}$$

$2-\sqrt{2}$

a

2013 online

If the image of point P(2, 3) in a line L is Q(4, 5) then the image of point R(0, 0) in the same line is
(a) (2, 2) (b) (4, 5) (c) (3, 4) (d) (7, 7)



P(2, 3), Q(4, 5)

slope of PQ $\frac{5-3}{4-2} = \frac{2}{2} = 1$

slope of RN $\frac{k-0}{h-0} = \frac{k}{h}$.

slope of PQ = slope of RN.

$1 = \frac{k}{h}$

$k = h$ I

Co-ordinate of M $= \left(\frac{4+2}{2}, \frac{5+3}{2}\right)$
$(3, 4)$

slope of PQ = 1

slope of L = -1

equation of L

$(y-4) = -(x-3)$

$y-4 = -x+3$

$x+y-7 = 0$ II

S is mid point of RN.

co-ordinates of $S = \left(\frac{0+h}{2}, \frac{0+k}{2}\right)$

$\left(\frac{h}{2}, \frac{k}{2}\right)$

It lies on the line II

$\frac{h}{2} + \frac{k}{2} - 7 = 0$

$h + k - 14 = 0$

From I

$h + h - 14 = 0$ $[\because k = h]$

$2h - 14 = 0$

$h = 7$, $k = 7$

$(7, 7)$, (d)

2013 online

Q. Let $A(-3,2)$ and $B(-2,1)$ be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x+4y+2=0$, then vertex C lies on the line (a) $4x+3y+5=0$ (b) $3x+4y+3=0$
(c) $4x+3y+3=0$ (d) $3x+4y+5=0$

sol $A(-3,2)$ $B(-2,1)$ $C(h,k)$

centroid $\left(\frac{-3-2+h}{3}, \frac{2+1+k}{3}\right)$

$\left(\frac{-5+h}{3}, \frac{3+k}{3}\right)$

which lies on $3x+4y+2=0$

$$3\left(\frac{-5+h}{3}\right)+4\left(\frac{3+k}{3}\right)+2=0$$

$$-15+3h+12+4k+6=0$$

$$3h+4k+3=0$$

$$3h+4k+3=0$$

Vertex $C(h,k)$ lies on the line $3x+4y+3=0$

(b)

2014 offline

Q. Let a, b, c, d be non zero numbers. If the point of intersection of the lines

$4ax + 2ay + c = 0$ and
$5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

(a) $2bc - 3ad = 0$ (b) $2bc + 3ad = 0$
(c) $3bc - 2ad = 0$ (d) $3bc + 2ad = 0$

Sol Let point of intersection be

$(h, -h)$

$(h, -h)$ satisfies $4ax + 2ay + c = 0$

$$4ah + 2a(-h) + c = 0$$
$$4ah - 2ah + c = 0$$
$$h(4a - 2a) = -c$$
$$2ah = -c$$
$$h = -\frac{c}{2a}$$

$(h, -h)$ satisfies $5bx + 2by + d = 0$

$$5bh + 2b(-h) + d = 0$$
$$5bh - 2bh + d = 0$$
$$h(5b - 2b) = -d$$
$$3bh = -d$$
$$h = -\frac{d}{3b}$$

$$-\frac{c}{2a} = -\frac{d}{3b}$$

$$-3bc + 2ad = 0.$$

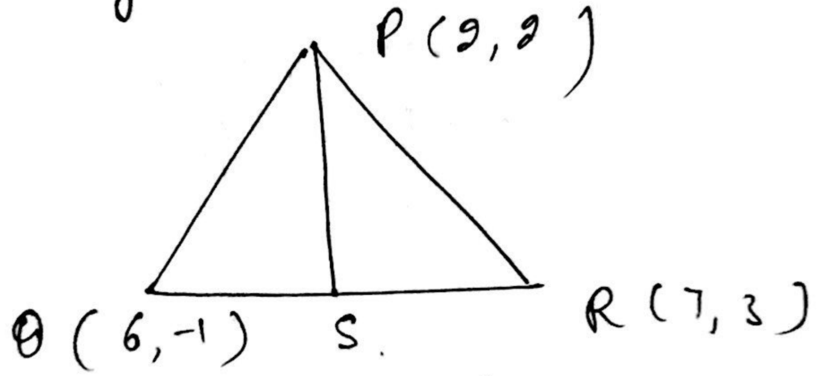
$$3bc - 2ad = 0$$ (c)

2014 off line

Q. Let PS be the median of the triangle with vertices $P(2,2)$ $Q(6,-1)$ & $R(7,3)$. The equation of the line passing $(1,-1)$ and parallel to PS is

(a) $4x-7y-11=0$ (b) $2x+9y+7=0$

(c) $4x+7y+3=0$ (d) $2x-9y-11=0$



Co-ordinate of S $\left(\frac{13}{2}, 1\right)$

Slope of PS $= \dfrac{2-1}{2-\frac{13}{2}} = -\dfrac{2}{9}$

slope of required line which is $\parallel$ to PS $= -\dfrac{2}{9}$

It passes through $(1,-1)$

$$y+1 = -\frac{2}{9}(x-1)$$

$$9y+9 = -2x+2$$

$$2x+9y+7=0$$

(b)

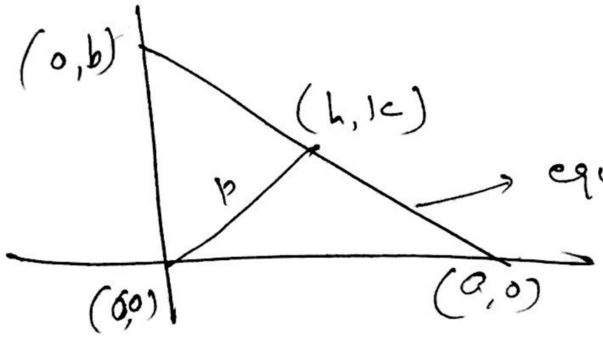
2014 online

Q. Let a and b be any two numbers satisfying $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$

Then the foot of perpendicular from the origin on the variable line $\frac{x}{a} + \frac{y}{b} = 1$ lies on

(a) a hyperbola with each semi axis $= \sqrt{2}$
(b) a hyperbola with each semi axis $= 2$
(c) a circle of radius $= 2$
(d) a circle of radius $= \sqrt{2}$

Sol.



equation of line $\frac{x}{a} + \frac{y}{b} = 1$

p is the length of perpendicular from origin on the line $\frac{x}{a} + \frac{y}{b} = 1$

$$p = \left| \frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{(1/a)^2 + \left(\frac{1}{b}\right)^2}} \right|$$

$$= \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \quad \cancel{\neq \frac{ab}{\sqrt{a^2}}}$$

$$= \frac{1}{\sqrt{\frac{1}{4}}} = 2.$$

$\sqrt{(0-h)^2 + (0-k)^2} = 2 \quad \sqrt{h^2+k^2} = 2 \quad h^2+k^2 = 4$

(h,k) lies on circle with radius 2 (c)