2011 offline

Consider the following relation R on the set of real square matrices of order 3

$$R = \{(A,B) : A = P^{-1}BP \text{ for some invertible matrix } P\}$$

Statement-1 : R is an equivalence relation

Statement-2 : For any two invertible 3×3 matrices M and N $(MN)^{-1} = N^{-1}M^{-1}$

R is reflexive as $A = I^{-1}AI$

$$\Rightarrow (A,A) \in R \text{ for all } A$$

R is symmetric as

$$A = P^{-1}BP$$

$$B = PAP^{-1}$$

$$B = (P^{-1})^{-1} A P^{-1}$$

$$\Rightarrow (B,A) \in R$$

R is trasitive as

Assume $(A,B) \in R$ and $(B,C) \in R$

$$A = P^{-1}BP \quad \text{and} \quad B = Q^{-1}CQ.$$

$$A = P^{-1}(Q^{-1}CQ)P$$

$$= (P^{-1}Q^{-1})C(QP)$$

$$= (QP)^{-1}C(QP)$$

$$\Rightarrow AC \in R$$

$\Rightarrow$ Statement I is true R is an equivalence relation

statement 2 II is also true

statement 2 is correct explanation of statement 1

2011 off line

Let f be a function defined by

$$f(x) = (x-1)^2 + 1 \quad (x \geq 1)$$

Statement 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement 2 : f is byection and

$$f^{-1}(x) = 1 + \sqrt{x-1} \qquad x \geq 1$$

Solution — $f(x) = (x-1)^2 + 1$

$$y = (x-1)^2 + 1$$

$$y - 1 = (x-1)^2$$

$$x - 1 = \sqrt{y-1}$$

$$x = 1 + \sqrt{y-1} \qquad y \geq 1$$

$$f^{-1}(x) = 1 + \sqrt{x-1}$$

Statement 2 is true

$f(x) = f^{-1}(x)$ given

$\cancel{1+\sqrt{x-1} = 1+\sqrt{x-1}}$

$$(x-1)^2 + \cancel{1} = \cancel{1} + \sqrt{x-1}$$

$$(x-1)^2 = \sqrt{x-1}$$

$$(x-1)^4 = x-1$$

$$(x-1)^4 - (x-1) = 0$$

$$(x-1)\left[(x-1)^3 - 1\right] = 0$$

$x - 1 = 0$ or $(x-1)^3 - 1 = 0$

$x = 1 \qquad (x-1)^3 = 1$

$$x - 1 = 1^{1/3}$$

$$x = 2$$

$x = \{1, 2\}$ statement 1 is true

statement 2 is correct explanation of statement 1

2011 offline

Let R be the set of real numbers

Statement-1 $A = \{(x,y) \in R \times R : y-x$ is an integer$\}$ is an equivalence relation on R

Statement-2 $B = \{(x,y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R

[a] Statement 1 is true Statement 2 is true Statement 2 is correct explanation of Statement 1

(b) Statement-1 is true Statement-2 is true Statement-2 is not correct explanation of Statement 1

(c) Statement 1 is true Statement-2 is false

(d) Statement-1 is false Statement-2 is true

For statement 1

It is reflexive as $x - x = 0$ is an integer

It is symmetric as $y - x$ is an integer
$x - y$ is an integer

It is transitive as $x - y$ is an integer
$y - z$ is an integer

Add them $x - \cancel{y} + \cancel{y} - z = x - z$ is an integer

$\Rightarrow (x, z) \in A$

So it is a equivalence relation on R

For statement 2

$x = \alpha y$

$\frac{x}{y} = \alpha$

$\frac{x}{y}$ is a rational number

$\frac{y}{x}$ may not be rational number

Example 0 is a rational number

$\frac{1}{0}$ is not rational number

B is not equivalence relation on R

(C)

2011 offline

Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$
Then the set of all x satisfying
$(fogogof)(x) = (gogof)(x)$ where
$(fog)(x) = f(g(x))$ is

(a) $\pm\sqrt{n\pi}$ $n \in \{0, 1, 2 \ldots\}$

(b) $\pm\sqrt{n\pi}$ $n \in \{1, 2 \ldots\}$

(c) $\frac{\pi}{2} + 2n\pi$ $n \in \{\ldots -2, -1, 0, 1, 2 \ldots\}$

(d) $2n\pi$ $n \in \{\ldots -2, -1, 0, 1, 2 \ldots\}$

$$f(x) = x^2, \quad g(x) = \sin x$$

$$(gof)(x) = g(f(x)) = g(x)^2 = \sin x^2$$

$$go(gof)(x) \cancel{=gof} = g(\sin x^2)$$
$$= \sin(\sin x^2)$$

$$(fogogof)(x) = f(\sin(\sin x^2))$$
$$= (\sin(\sin x^2))^2$$

$$(gogof)(x) = gog(f(x)) = gog(x^2)$$
$$= g(g(x^2))$$
$$= g(\sin x^2)$$
$$= \sin(\sin x^2)$$

$$((f \circ g \circ g \circ f)(x)) = (g \circ g \circ f)(x$$

$$(\sin(\sin x^2))^2 = \sin(\sin x$$

$$(\sin(\sin x^2))^2 - \sin(\sin x^2)$$

$$\sin(\sin x^2)[\sin(\sin x^2) - 1$$

$\sin(\sin x^2) = 0$ or $\sin(\sin$

$\sin x^2 = 0$

$x^2 = n\pi$

$x = \pm\sqrt{n\pi}$

$\sin(\sin$

$\sin x$

which

$|$

as $-1 \leq$

2012

The function $f : [0,3] \to [1,29]$ defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is
(a) one one and onto (b) onto but not one one
(c) one one but not onto (d) neither one one nor onto

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$
$$f'(x) = 6x^2 - 30x + 36$$
$$= 6(x^2 - 5x + 6)$$
$$= 6(x-2)(x-3)$$

$+ \quad - \quad +$

$2 \quad 3$

Domain $[0,3]$
In this domain function is increasing as well as decreasing
So it is not one one

Put $f'(x) = 0$
$6(x-2)(x-3) = 0 \Rightarrow x = 2, 3$
$f(x) = 2x^3 - 15x^2 + 36x + 1$
$f(0) = 1 \quad f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 1$
$= 29$
$f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1 = 28$
Range $\in [1, 29] \Rightarrow$ Range $=$ Codomain
So onto

(b)

2013 offline

Let A and B be two sets containin
and 4 elements respectively. The
of subsets of A × B having 3 or
elements is

(a) 211 (b) 256 (c) 220 (d)

$n(A) = 2 \quad n(B) = 4$

$n(A \times B) = n(A)\, n(B) = 2 \times$

Total number of subsets of A × B

Number of subsets of A × B hav
3 or more elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 256 - (1 + 8 + 28)$

$= 256 - 37$

$= 219$ (d)

2014

A relation on the set $A = \{x : |x| <$
where Z is the set of integers
defined by $R = \{(x, y) : y = |x|$
Then the number of elements
power set of R is

(a) 32 (b) 16 (c) 8 (d) 64.

$A = \{x : |x| < 3, x \in Z\}$

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(x, y) : y = |x|$ $x \neq$

$= \{(-2, 2), (0, 0), (1, 1), ($

R has four elements

Number of elements in power set $= 2^4 = 16$

(b)

2014 offline

Q. $f_k(x) = \frac{1}{K}(\sin^k x + \cos^k x)$ where $x \in$ [cut off]
and $k \geq 1$ then $f_4(x) - f_6(x)$ is equal [cut off]

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

$$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x) \quad k \geq 1, \; x \in$$

$f_4(x) - f_6(x)$

$\frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$\frac{1}{4}(1 - 2\sin^2 x\cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x \cos^2 x)$

$\frac{1}{4} - \frac{\cancel{2}}{\cancel{4}_2}\sin^2 x\cos^2 x - \frac{1}{6} + \frac{\cancel{3}}{\cancel{6}_2}\sin^2 x \cos^2 x$

$\frac{1}{4} - \cancel{\frac{1}{2}\sin^2 x\cos^2 x} - \frac{1}{6} + \cancel{\frac{1}{2}\sin^2 x\cos^2 x}$

$\frac{1}{4} - \frac{1}{6}$

$\frac{3-2}{12} = \frac{1}{12}$

d

2014

Let $f : R \rightarrow R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$

Then f is

(a) onto but not one one (b) both one one and onto

(c) one one but not onto (d) neither one one nor onto

$$f(x) = \frac{|x|-1}{|x|+1}$$

Put $x = -1$ $\quad f(-1) = 0$

$x = 1$ $\quad f(1) = 0$

$\therefore$ f is not one one

$$f(x) = \frac{|x|-1}{|x|+1} = \frac{|x|-1+1-1}{|x|+1}$$

$$\frac{|x|+1-2}{|x|+1} = \frac{\cancel{|x|+1}}{\cancel{|x|+1}} - \frac{2}{|x|+1}$$

$$= 1 - \frac{2}{|x|+1}$$

$\Rightarrow f(x) = 1 - \frac{2}{|x|+1} \leq 1$ for all $x \in R$

Range $(-\infty\ 1]$

$\Rightarrow$ Co-domain $\neq$ Range

So it is not onto

So neither one one nor onto

(d)

2014

Let P be the relation defined on the set of all real numbers such that $P = \{(a,b) : \sec^2 a - \tan^2 b = 1\}$ Then P is

(a) reflexive and symmetric but not transitive

(b) reflexive and transitive but not symmetric

(c) symmetric and transitive but not reflexive

(d) an equivalence relation.

$$P = \{(a,b) : \sec^2 a - \tan^2 b = 1\}$$

It is not reflexive as $\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \notin P$

$$\because \sec^2 \frac{\pi}{2} - \tan^2 \frac{\pi}{2} \neq 1$$

P is symmetric

Suppose $(a,b) \in P$

$$\sec^2 a - \tan^2 b = 1$$

$$(1 + \tan^2 a) - (\sec^2 b - 1) = 1$$

$$1 + \tan^2 a - \sec^2 b + 1 = 1$$

$$2 + \tan^2 a - \sec^2 b = 1$$

$$\sec^2 b - \tan^2 a = 1$$

$$\Rightarrow (b,a) \in P$$

Hence symmetric

P is transitive

Suppose $(a,b) \in P \Rightarrow \sec^2 a - \tan^2 b = 1$ I

$(b,c) \in P \Rightarrow \sec^2 b - \tan^2 c = 1$ II

Add I & II $\quad \sec^2 a - \tan^2 b + \sec^2 b - \tan^2 c = 2$

$$\sec^2 a - \cancel{\tan^2 b} + 1 + \cancel{\tan^2 b} - \tan^2 c = 2$$

(c) $\quad \sec^2 a - \tan^2 c = 1 \Rightarrow (a,c) \in P$

2015 online

In a certain town 25% of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements

(a) 5% families own both a car and a phone
(b) 35% families own either a car or a phone
(c) 40000 families live in town Then

(a) only a and b are correct.
(b) only a and c are correct.
(c) only b and c are correct.
d) all (a), (b), and (c) are correct.

Let the total number of families $= x$

$n(P) = 25\% = 0.25x$

$n(C) = 15\% = 0.15x$

$n(P' \cap C') = 65\% = 0.65x$

$n(P \cup C) = x - n(P' \cap C')$

$= x - 0.65x$

$= 0.35x$

$\Rightarrow$ 35% of the families own either a car or a phone

$$n(P \cup C) = n(P) + n(C) - n(P \cap C)$$

$$0.35x = 0.25x + 0.15x - n(P \cap C)$$

$$0.35x - 0.25x - 0.15x = -n(P \cap C)$$

$$-0.05 = -n(P \cap C)$$

$$n(P \cap C) = 0.05$$

$\Rightarrow$ 5% of the families own a car and a phone

$$0.05x = 2000$$

$$\frac{5}{100}x = 2000$$

$$x = \cancel{2000}^{400} \times \frac{100}{\cancel{5}}$$

$$= 40{,}000$$

a , b , c

2015 offline

Let A and B be two sets containing
two elements respectively Then the
subsets of the set $A \times B$ each having
least three elements is

(a) 219 (b) 256 (c) 275 (d) 5

$$n(A) = 4$$

$$n(B) = 2$$

$$n(A \times B) = 4 \times 2 = 8$$

Number of subsets of $A \times B$ which
at least three elements

$$^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C$$

$$2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$$

$$2^8 - (1 + 8 + 28)$$

$$256 - 37$$

$$219$$

(a)

2016 offline

If the function $f : [1, \infty) \to [1, \infty)$ is by $f(x) = 3^{x(x-1)}$ Then $f^{-1}(x)$ is

(a) $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_3 x}\right)$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4 \right.$

(c) not defined (d) $\left(\frac{1}{3}\right)^{x(x-1)}$

$$f(x) = 3^{x(x-1)}$$

$$y = 3^{x(x-1)} \qquad y \geq 1$$

$$\log_3 y = x(x-1)$$

$$\log_3 y = x^2 - x$$

$$x^2 - x - \log_3 y = 0$$

$$x = \frac{1 \pm \sqrt{1 + 4\log_3 y}}{2}$$

As $x \geq 1$

$$x = \frac{1 + \sqrt{1 + 4\log_3 y}}{2}$$

$$f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_3 x}}{2} = \frac{1}{2}\left(1 + \sqrt{1 \right.$$

(b)

2016

If $f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad x \neq 0$

and $S = \{x \in R : f(x) = f(-x)\}$ then S

(a) is an empty set.

(b) contains exactly one element

(c) contains exactly two elements

(d) contains more than two elements

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x \qquad \text{I}$$

Replace x by $\frac{1}{x}$

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \qquad \text{II}$$

Write I − 2(II)

$$f(x) + 2f\left(\frac{1}{x}\right) - 2\left(f\left(\frac{1}{x}\right) + 2f(x)\right) = 3x - \frac{6}{x}$$

$$f(x) + \cancel{2f\left(\frac{1}{x}\right)} - \cancel{2f\left(\frac{1}{x}\right)} - 4f(x) = 3x - \frac{6}{x}$$

$$-3f(x) = 3x - \frac{6}{x}$$

$$f(x) = -x + \frac{2}{x}$$

$$f(x) = \frac{2}{x} - x$$

$$f(x) = f(-x)$$

$$\frac{2}{x} - x = -\frac{2}{x} + x$$

$$\frac{4}{x} = 2x$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

S has exactly two elements

(c)

2017 offline

The function $f : R \to [-\frac{1}{2}, \frac{1}{2}]$ defined by $f(x) = \frac{x}{1+x^2}$ is

(a) invertible (b) injective but not surjective

(c) surjective but not injective

(d) neither injective nor surjective

$$f(x) = \frac{x}{1+x^2}$$

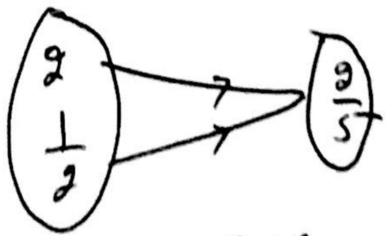
Take $x = 2$ and $x = \frac{1}{2}$

$$f(2) = \frac{2}{1+2^2} = \frac{2}{5}$$

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2} = \frac{\frac{1}{2}}{1+\frac{1}{4}} = \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{5} = \frac{2}{5}$$

$$f(2) = f\left(\frac{1}{2}\right)$$

$\Rightarrow$



$\Rightarrow$ f is not one one.

$$f(x) = \frac{x}{1+x^2}$$

$$y = \frac{x}{1+x^2}$$

$$y(1+x^2) = x$$

$$y + yx^2 = x$$

$$x^2 y - x + y = 0$$

$$x = \frac{+1 \pm \sqrt{(-1)^2 - 4y^2}}{2y}$$

$$= \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$1-4y^2 \geq 0$$

$$1 \geq 4y^2$$

$$4y^2 \leq 1$$

$$y^2 \leq \frac{1}{4}$$

$$-\frac{1}{2} \leq y \leq \frac{1}{2}.$$

Range $[-\frac{1}{2}, \frac{1}{2}]$

Range = co-domain

so it is onto

f(x) is surjective not injecti

2017 offline

Let $a, b, c \in R$ & $f(x) = ax^2 + bx + c$ be such that $a+b+c = 3$ and

$$f(x+y) = f(x) + f(y) + xy \quad \forall\, x, y \in R$$

Then $\sum_{n=1}^{10} f(n)$ is equal to

a) 330 (b) (165 (c) 190 (d) 255

$$f(x+y) = f(x) + f(y) + xy$$

Put $y = 0$

$$f(x) = f(x) + f(0) + 0$$

$$f(x) - f(x) = f(0)$$

$$0 = f(0) \Rightarrow f(0) = 0$$

$$f(x) = ax^2 + bx + c$$

$$f(0) = a \cdot 0 + b \cdot 0 + c$$

$$0 = c \Rightarrow c = 0$$

Now Put $y = -x$

$$f(x+y) = f(x) + f(y) + xy$$

$$f(x-x) = f(x) + f(-x) - x \cdot x$$

$$f(0) = ax^2 + \cancel{bx} + c + ax^2 - \cancel{bx} + c - x^2$$

$$0 = 2ax^2 - x^2$$

$$0 = x^2(2a - 1) \Rightarrow a = \frac{1}{2}$$

$$a + b + c = 3$$

$$\frac{1}{2} + b + 0 = 3$$

$$b = 3 - \frac{1}{2}$$

$$b = \frac{5}{2}$$

$$f(x) = ax^2 + bx + c$$

$$= \frac{1}{2}x^2 + \frac{5}{2}x + 0$$

$$f(x) = \frac{x^2 + 5x}{2}$$

$$f(n) = \frac{n^2 + 5n}{2}$$

$$\sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + 5n.$$

$$= \frac{1}{2}\left[\sum_{n=1}^{10} n^2 + \sum_{n=1}^{10} 5n\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{10}^{5} \times 11 \times \cancel{21}^{7}}{\cancel{6}\,\cancel{2}} + 5 \times \frac{\cancel{10}^{5} \times 11}{\cancel{2}}\right]$$

$$= \frac{1}{2}\left[385 + 275\right]$$

$$= \frac{1}{\cancel{2}} \times \cancel{660}^{330} = 330$$

a