2011 offline

Q A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving in ₹ 40 more than the saving of the immediately previous month. His total saving from the start of the service will be ₹ 11040 after

(a) 21 months (b) 18 months (c) 19 months
(d) 20 months.

Sol. Total saving = 11040 ₹

First three months savings are
200, 200, 200

After that 240, 280 .....

$\Rightarrow 200 + 200 + 200 + 240 + 280 \quad n \text{ months}$

11040

$200 + 240 + 280 \ldots (n-2) \text{ months} = 11040 - 400$

$200 + 240 + 280 \ldots (n-2) \text{ months} = 10640$

$\frac{n-2}{2}\left(2 \times 200 + (n-2-1)\, 40\right) = 10640$

$$\frac{n-2}{2}\left(400+(n-3)40\right)=10640$$

$$(n-2)(400+40n-120)=10640\times 2$$

$$(n-2)(40n+280)=21280$$

$$40n^2+280n-80n-560=21280$$

$$40n^2+200n-21280=0$$

$$n^2+5n-546=0$$

$$n^2+26n-21n-546=0$$

$$(n+26)(n-21)=0$$

$$n=-26,\ 21$$

$$n=21 \text{ as } n>0$$

(a)

2011 offline

Q. Let a_n be the nth term of A.P

If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$ the

common difference of the A.P is

(a) $\alpha - \beta$ (b) $\frac{1}{100}(\alpha - \beta)$ (c) $(\beta - \alpha)$

(d) $\frac{1}{200}(\alpha - \beta)$

Sol $\alpha = a_2 + a_4 + a_6 \ldots \ldots a_{200}$

$\beta = a_1 + a_3 + a_5 \ldots \ldots a_{199}$

$\alpha - \beta = (a_2 - a_1) + (a_4 - a_3) \ldots (a_{200} - a_{199})$

$\alpha - \beta = \sum_{r=1}^{100} d$

$\alpha - \beta = 100d$

$\frac{1}{100}(\alpha - \beta) = d$

b

2012 off line

Q. If 100 times the 100th term of
A.P with non zero common differe
equals the 50 times its 50th
then the 150 term of this AP

(a) 150 time its 50th term

(b) 150 (c) zero (d) −150

So $100\, a_{100} = 50\, a_{50}$

$$100\, a_{100} - 50\, a_{50} = 0$$

$$2\, a_{100} - a_{50} = 0$$

$$2(a+99d) - (a+49d) = 0$$

$$2a + 198d - a - 49d = 0$$

$$a + 149d = 0$$

$$a_{150} = 0$$

(c)

2012

Q. Statement I

The sum of the series

$1 + (1+2+4) + (4+6+9) + (9+12+16)$

$\dots \dots (361 + 380 + 400)$ is 800

Statement II

$$\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$$ for any natural number n

Sol. Statement I is false

Statement II is true

$$\because \sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$$

$$\left(1^3 - (1-1)^3\right) + \left(2^3 + (2-1)^3\right) + \left(3^3 + (3-1)^3\right) + \dots \dots = n^3$$

$$(\cancel{1}^3 - 0^3) + (\cancel{2}^3 + \cancel{1}^3) + (3^3 - \cancel{2}^3) \dots \dots$$

$$\dots \dots \left(n^3 - (\cancel{n-1})^3\right) = n^3.$$

$$n^3 = n^3$$

which is true

$$\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$$

$$\sum_{k=1}^{n} \left(k-(k-1)\right)\left(k^2 + k(k-$$

$$\sum_{k=1}^{n} \left(k^2 + k(k-1) + (k-1)^2\right.$$

$$\sum_{k=1}^{n} (k-1)^2 + k(k-1) + k$$

Put $n = 20$.

$1 + (1+2+4) + (4+6+$

$\ldots\ldots \left(361 + 380 + 40\right.$

80

$\Rightarrow$ Statement I is f

2012

Q. 27. The line $2x+y=k$ passes through the point which divides the segment joining the points $(1,1)$ and $(2,4)$ in the ratio $3:2$, then k equals

(a) 6 (b) 11/5 (c) 29/5 (d) 5

$$(1,1) \overset{3 \; : \; 2}{\underset{P(x,y)}{\longrightarrow}} (2,4)$$

$$x = \frac{3\times 2 + 2\times 1}{3+2} = \frac{6+2}{5} = \frac{8}{5}$$

$$y = \frac{3\times 4 + 2\times 1}{5} = \frac{12+2}{5} = \frac{14}{5}$$

$\left(\frac{8}{5}, \frac{14}{5}\right)$ lies on the line $2x+y=k$

$$2\left(\frac{8}{5}\right) + \frac{14}{5} = k$$

$$\frac{16}{5} + \frac{14}{5} = k$$

$$6 = k$$

(a)

2013 online

Q. Let $a_1, a_2, a_3 \ldots$ be an AP such that

$$\frac{a_1 + a_2 \ldots a_p}{a_1 + a_2 \ldots a_q} = \frac{p^2}{q^2} \quad p \neq q$$

Then $\frac{a_6}{a_{21}}$ is equal to

(a) $\frac{41}{11}$ (b) $\frac{121}{168}$ (c) $\frac{11}{41}$ (d) $\frac{121}{1861}$

$$\frac{a_1 + a_2 \ldots a_p}{a_1 + a_2 \ldots a_q} = \frac{p^2}{q^2}$$

$$\frac{\frac{p}{2}(2a_1 + (p-1)d)}{\frac{q}{2}(2a_1 + (q-1)d} = \frac{p^2}{q^2}$$

$$\frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\frac{a_1 + \frac{1}{2}(p-1)d}{a_1 + \frac{1}{2}(q-1)d} = \frac{p}{q}$$

$\frac{1}{2}(p-1) = 5 \quad p-1 = 10, p = 11$

$\frac{1}{2}(q-1) = 20 \quad (q-1) = 40 \quad q = 41$

if $p = 11, q = 41$

$$\frac{a_1 + 5d}{a_1 + 20d} = \frac{p}{q} = \frac{11}{41}$$

$\frac{a_6}{a_{21}} = \frac{11}{41}$ (c)

2017 online

Q Given sum of first n terms of an AP is $2n + 3n^2$. Another AP is formed with the same first term and is double the common difference, the sum of n terms of the new AP is

(a) $n + 4n^2$ (b) $6n^2 - n$ (c) $n^2 + 4n$ (d) $3n + 2n^2$

Sol $S_n = 2n + 3n^2$

$S_1 = 5 \quad S_2 = 16$

$a_1 = 5 \quad a_2 = S_2 - S_1 = 16 - 5 = 11$

$d = 11 - 5 = 6$

$S_n = ?$ if $a = 5 \quad d = 2 \times 6 = 12.$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$= \frac{n}{2}\left(2 \times 5 + (n-1)12\right)$$

$$= \frac{n}{2}\left(10 + 12n - 12\right)$$

$$= \frac{n}{2}\left(12n - 2\right)$$

$$= \frac{n}{\cancel{2}} \times \cancel{2}\left(6n - 1\right)$$

$$= 6n^2 - n$$

(b)

2013 ~~onbroct~~

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} +$ upto 11 term

(a) $\frac{7}{2}$ (b) $\frac{11}{4}$ (c) $\frac{11}{2}$ (d) $\frac{60}{11}$

$$a_n = \frac{2n+1}{1^2+2^2+3^2 \cdots n^2}$$

$$= \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{\cancel{(2n+1)} \times 6}{n(n+1)\cancel{(2n+1)}}$$

$$= \frac{6}{n(n+1)}$$

$$= 6\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\sum_{n=1}^{11} a_n = 6\left[\left(1-\cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}}-\frac{1}{4}\right) + \cdots \left(\frac{1}{11}-\frac{1}{12}\right)\right]$$

$$= 6\left[1 - \frac{1}{12}\right]$$

$$= \cancel{6} \times \frac{11}{\cancel{12}_2} = \frac{11}{2}$$

(c)

2013 online

Q. The value of $1^2+3^2+5^2 \cdots\cdots 25^2$ is

(a) 2925 (b) 1469 (c) 1728 (d) 1456

Sol $1^2+3^2+5^2 \cdots\cdots 25^2$

$$\sum_{r=1}^{13} (2r-1)^2$$

[$1+3+5 \cdots\cdots 25$

$a_n = a+(n-1)d$

$25 = 1+(n-1)2$

$12 = n-1 \Rightarrow n=13$]

$$\sum_{r=1}^{13} (4r^2+1-4r)$$

$$4\left[\frac{n(n+1)(2n+1)}{6}\right]_{n=13} + 13 + 4\left[\frac{n(n+1)}{2}\right]$$

$$\frac{\cancel{4}^{2} \times 13 \times 14 \times \cancel{27}^{9}}{\cancel{6}\,\cancel{3}} + 13 - \frac{\cancel{4}^{2} \times 13 \times 14}{\cancel{2}}$$

2925

(a)

2013 online

Q. The sum of the series

$2^2 + 2(4^2) + 3(6^2) + \ldots$ upto 10 terms

(a) 11300 (b) 11900 (c) 12100 (d) 12300

sol. $1 \cdot 2^2 + 2 \cdot 4^2 + 3 \cdot 6^2 \ldots$ upto 10 term

$$\sum_{k=1}^{10} k \cdot (2k)^2$$

$$\sum_{k=1}^{10} (k)(4k^2)$$

$$\sum_{k=1}^{10} 4k^3$$

$$4\left(\frac{k(k+1)}{2}\right)^2_{k=10}$$

$$4\left(\frac{\cancel{10}^{\,5} \times 11}{\cancel{2}}\right)^2$$

4×3025

12100

c

2013 online

Q. 9. If $a_1, a_2, a_3 \ldots a_n$ are in A.P.

$a_4 - a_7 + a_{10} = m$ Then sum of

13 terms of the AP is

(a) 10m (b) 12m (c) 13m (d) 15

$$a_4 - a_7 + a_{10} = m$$

$$a + 3d - (a + 6d) + a + 9d = m$$

$$a + 3d - \cancel{a} - 6d + \cancel{a} + 9d = m$$

$$a + 6d = m$$

$$S_{13} = \frac{13}{2}\left(2a + (13-1)d\right)$$

$$= \frac{13}{2}\left(2(a + 6d)\right)$$

$$= \frac{13}{\cancel{2}} \times \cancel{2}m$$

$$= 13m$$

(c)

2013 off line

Q. If x, y, z are in A.P and $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are also in A.P

Then (a) $2x = 3y = 6z$

(b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$

(d) $x = y = z$

Sol x, y, z are in A P $\Rightarrow 2y = x + z$

$\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are in A.P $\Rightarrow$

$$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$$

$$\tan^{-1}\frac{2y}{1-y^2} = \tan^{-1}\frac{x+z}{1-xz}$$

$$\frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$$\frac{2y}{1-y^2} = \frac{2y}{1-xz} \quad [\because 2y = x+z]$$

$$\cancel{1} - y^2 = \cancel{1} - xz$$

$$y^2 = xz$$

$$\left(\frac{x+z}{2}\right)^2 = xz \Rightarrow (x-z)^2 = 0$$

$\Rightarrow x = z$ So $2y = 2x \Rightarrow y = x$

$\Rightarrow x = y = z$ (d)

2013 offline

Q The sum of first 20 terms of the sequence 0.7, 0.77, 0.777 .... is

(a) $\frac{7}{9}(99-10^{-20})$ (b) $\frac{7}{81}(179+10^{-20})$

(c) $\frac{7}{9}(99+10^{-20})$ (d) $\frac{7}{81}(179-10^{-20})$

Sol $S = 0.7 + 0.77 + 0.777 \cdots$

$= \frac{7}{9}(.9 + .99 + .999 \cdots 20 \text{ terms})$

$= \frac{7}{9}\left((1-0.1)+(1-0.01)+(1-0.001)\cdots 20 \text{ terms}\right)$

$= \frac{7}{9}\left[20-(0.1+0.01+0.001\cdots 20 \text{ terms})\right]$

$= \frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right]$

$= \frac{7}{9}\left[20-\frac{\frac{1}{10}(1-10^{-20})}{\frac{9}{10}}\right]$

$= \frac{7}{9}\left[20-\frac{1}{\cancel{10}}\times\frac{\cancel{10}}{9}(1-10^{-20})\right]$

$= \frac{7}{9}\left[20-\frac{1}{9}+\frac{10^{-20}}{9}\right]$

$= \frac{7}{9}\left[\frac{179}{9}+\frac{10^{-20}}{9}\right]$

$\frac{7}{81}(179+10^{-20})$ (b)

2013 online

Q) If $S = \tan^{-1}\left(\frac{1}{n^2+n+1}\right) + \tan^{-1}\left(\frac{1}{n^2+3n+3}\right)$

$+ \cdots\cdots \tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$

Then $\tan S$ is equal to:

(a) $\frac{20}{401+20n}$ (b) $\frac{n}{n^2+20n+1}$ (c) $\frac{20}{n^2+20n+1}$

(d) $\frac{n}{401+20n}$

Sol. $t_k = \tan^{-1}\left(\frac{1}{1+(n+k-1)(n+k)}\right)$

$= \tan^{-1}\left(\frac{(n+k)-(n+k)+1}{1+(n+k-1)(n+k)}\right)$

$= \tan^{-1}\left(\frac{(n+k)-(n+k-1)}{1+(n+k-1)(n+k)}\right)$

$= \tan^{-1}(n+k) - \tan^{-1}(n+k-1)$

$S = \sum_{k=1}^{20} t_k = \left(\cancel{\tan^{-1}(n+1)} - \tan^{-1}n\right) +$

$\left(\cancel{\tan^{-1}(n+2)} - \cancel{\tan^{-1}(n+1)}\right) + \cdots\cdots$

$\cdots\cdots \left(\tan^{-1}(n+20) - \cancel{\tan^{-1}(n+19)}\right)$

$= \tan^{-1}(n+20) - \tan^{-1}n$

$$\tan^{-1}\left(\frac{\cancel{n}+20-\cancel{n}}{1+n(n+20)}\right)$$

$$S = \tan^{-1}\frac{20}{1+n(n+20)}$$

$$\tan S = \frac{20}{1+n^2+20n}$$

$$= \frac{20}{n^2+20n+1}$$

c

2014 online

Q. If the sum $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots$ upto 20 terms is equal to $\frac{k}{21}$. Then k is equal to (a) 120 (b) 180 (c) 240 (d) 60

$$a_k = \frac{2k+1}{1^2+2^2+\ldots k^2}$$

$$= \frac{2k+1}{\frac{k(k+1)(2k+1)}{6}}$$

$$= \frac{6}{k(k+1)}$$

$$= 6\left[\frac{1}{k} - \frac{1}{k+1}\right]$$

$$\sum_{k=1}^{20} a_k = 6\left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) \ldots \left(\frac{1}{20}-\frac{1}{21}\right)\right]$$

$$= 6\left[1 - \frac{1}{21}\right]$$

$$= 6 \times \frac{20}{21}$$

$$= \frac{120}{21}$$

$$\frac{120}{21} = \frac{k}{21}$$

$$\Rightarrow k = 120$$

(a)

2014 online

Q. The sum of first 20 terms comm[on]
the series

3+7+11+15+19+23+27+31 ...
1+6+11+16+21+26+31

(a) 4000 (b) 4020 (c) 4200 (d) 42[..]

Sol. common terms are

11, 31

a = 11 d = 20

$$S_{20} = \frac{20}{2}\left[2\times 11 + (20-1)\,20\right]$$

$$= 10\left[22 + 19\times 20\right]$$

$$= 10\left[22 + 380\right]$$

$$= 10\times 402$$

$$= 4020$$

(b)

2014 online

Q. The least positive integer n such that

$$1-\frac{2}{3}-\frac{2}{3^2}-\cdots-\frac{2}{3^{n-1}}<\frac{1}{100}$$

is (a) 4 (b) 5 (c) 6 (d) 7

$$1-\frac{2}{3}-\frac{2}{3^2}-\cdots-\frac{2}{3^{n-1}}$$

$$1-\left[\frac{2}{3}+\frac{2}{3^2}+\cdots+\frac{2}{3^{n-1}}\right]<\frac{1}{100}$$

$$1-\left[\frac{\frac{2}{3}\left(1-\left(\frac{1}{3}\right)^{n-1}\right)}{1-\frac{1}{3}}\right]<\frac{1}{100}$$

$$1-\left[\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{2}}\left(1-\left(\frac{1}{3}\right)^{n-1}\right)\right]<\frac{1}{100}$$

$$1-\left[1-\left(\frac{1}{3}\right)^{n-1}\right]<\frac{1}{100}$$

$$\cancel{1}-\cancel{1}+\left(\frac{1}{3}\right)^{n-1}<\frac{1}{100}$$

$$\left(\frac{1}{3}\right)^{n-1}<\frac{1}{100}$$

$$3^{n-1}>100$$

$$n-1\geq 5$$

$$n\geq 6$$

The least value is 6 (c)