2011 offline

There are 10 points in a plac
out of those 6 are collinear.
is the number of triangles for
Joining these points then

(a) $N \leq 100$ (b) $100 < N \leq 140$

(c) $140 < N \leq 190$ (d) $N > 190$

No of required triang

$$^{10}C_3 - {}^{6}C_3$$ [There a
colline

$$\frac{10!}{7!\times 3!} - \frac{6!}{3!\times 3!}$$

$$\frac{10\times\overset{3}{\cancel{9}}\times\overset{4}{\cancel{8}}}{\cancel{3}\times\cancel{2}} - \frac{\overset{2}{\cancel{6}}\times 5\times 4}{\cancel{3}\times\cancel{2}}$$

$$120 - 20$$

$$100$$

2011 off line

Statement I The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_3$

Statement II The number of choosing 3 places from 9 different places is ${}^{9}C_3$

We know the number of ways of distributing n items among r persons such that each person receives at least one item is

$${}^{n-1}C_{r-1}$$

Here in first statement the number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty (each box has at least one ball)

$${}^{10-1}C_{4-1} = {}^{9}C_3$$

Clearly statement II is True but it is not a correct explanation of first statement

2012 off line

The total number of ways in which 5 balls of different colours can be distributed among three persons so tha each person gets at least one ball is

(a) 75 (b) 150 (c) 210 (d) 243

Number of balls = 5

Number of persons = 3

~~We~~ It is given each person gets at least one ball

There are two cases

Case I (1, 1, 3)

1st person, Second person, Third person

$^5C_1$ ways × $^4C_1$ ways × $^3C_3$ ways

5 × 4 × 1 = 20 ways

They can be arranged among Themselv in 3 ways

Total number of ways = 20×3 = 60

Case II (2, 2, 1)

1st person, 2nd person, Third Person.

$^5C_2 \times {}^3C_2 \times {}^1C_1$

10 × 3 × 1 = 30 ways.

They can be arranged among themself in 3 ways

Total number of ways
$30 \times 3 = 90$

Total number of ways

$60 + 90 \pm 150$

ⓑ

2012 offline

Assuming the balls to be identical exce
for difference in colors, the number of
ways in which one or more balls can
selected from 10 white, 9 green and 7
black balls is

(a) 629 (b) 630 (c) 879 (d) 880

We are to find number of ways in
which one or more balls can be selecte

Number of ways to choose white balls = 10
[∵ no white ball or mor
white balls]

Number of ways to choose green balls = 9+
[∵ no green ball or more g

Number of ways to choose black balls = 7+
[∵ no black ball or more black

Number of ways to choose zero or more balls
any colour = 11 × 10 × 8 = 880

No of ways to choose one or more
balls = 880 − 1 = 879
[∵ Number of ways to choose zero

(c)

2013 online

On the sides AB, BC, CA of a triangle ABC 3, 4, 5 distinct point (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are

(a) 210 (b) 205 (c) 215 (d) 220

On AB there are 3 distinct points

On BC there are 4 distinct pts

On CA there are 5 distinct pts

Take one vertex on each

No of triangles will b

$$^3C_1 \times {}^4C_1 \times {}^5C_1 = 3$$

$$=$$

Take two vertices on a

and one from remaing

This can be done in

$$^3C_2 \times {}^9C_1 + {}^4C_2 \times {}^8C_1 + {}^5C$$

$$27 + 48 + 70$$

$$145$$

No of required tria

$$60 + 145 = 205$$

ⓒ

2013 online

5-digit numbers are formed using 2, 3, 5, 7, 9 without repeating the digit

If p is the number of such number that exceeds 20000 and

q is the number of those that lie between 30000 and 90000

Then p : q is

(a) 6 : 5 (b) 3 : 2 (c) 4 : 3 (d) 5 : 3

p : is the number that is formed by 2, 3, 5, 7, 9 exceed 20000 is 5!

q is the number that begins with 3, 5, or 7 lies between 30000 and 90000

$q = 3 \times 4!$

$$\frac{p}{q} = \frac{5!}{3 \times 4!} = \frac{5}{3}$$

$p : q = 5 : 3$

(d)

2013 online

A committee of 4 persons is to be form
from 2 ladies, 2 old men, and 4 youngme
such that it includes at least 1 lad
at least one old man and at
most 2 youngmen. Then the total
number of ways in which committee
can be formed is

(a) 40 (b) 41 (c) 16 (d) 32

Different number of ways of formin
committee are

I (1 lady, 1 old man, 2 young men)

II (1 lady, 2 old men, 1 young man)

III (2 ladies, 1 old man, 1 young man)

Required number of ways

$${}^2C_1 \times {}^2C_1 \times {}^4C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 +$$

$${}^2C_2 \times {}^2C_1 \times {}^4C_1$$

$$= 24 + 8 + 8 = 40$$

ⓐ

2013 off line

Let $T_n$ be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$ Then value of n

(a) 7 (b) 5 (c) 10 (d) 8

$$T_n = {}^nC_3 \quad \text{(given)}$$

$$T_{n+1} = {}^{n+1}C_3$$

$$T_{n+1} - T_n = 10 \quad \text{(given)}$$

$${}^{n+1}C_3 - {}^nC_3 = 10$$

$${}^nC_2 + \cancel{{}^nC_3} - \cancel{{}^nC_3} = 10$$

$$\left[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}\right]$$

$${}^nC_2 = 10$$

$$\frac{n!}{(n-2)!\,2!} = 10$$

$$n(n-1) = 20$$

$$n^2 - n - 20 = 0$$

$$(n-5)(n+4) = 0$$

$n = 5$ ⓑ

2014 online

8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do not occupy odd places is

(a) 160 (b) 120 (c) 60 (d) 48

1, 1, 2, 2, 2, 3, 4, 4.

There are 3 odd numbers 1, 1, 3

There are 5 even numbers 2, 2, 2, 4, 4

There are 4 odd places in 8-digit number

We are going to fill these odd places with even numbers and even places with the remaining numbers.

There are two cases

Case I let odd places filled by 2, 2, 4, 4 and even places filled by 1, 1, 3, 2

No of ways will be

$$\frac{4!}{2! \times 2!} \times \frac{4!}{2!}$$

$$= \frac{4 \times 3}{\cancel{2}} \times \frac{4 \times 3}{1} = 72 \text{ ways}$$

Case II odd places filled

2, 2, 2, 4 and even plac

filled by 1, 1, 3, 4

No of ways will be.

$$\frac{4!}{3!} \times \frac{4!}{2!}$$

$$4 \times 12 = 48 \text{ ways}$$

Total number of ways

$$72 + 48 = 120 \text{ wa}$$

(b)

2014 online

Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by ~~the~~ 66, then the number of men who participated in the tournament lies in the interval

(a) [8, 9] (b) [10, 12) (c) (11, 13]

(d) (14, 1)

Let number of men = $x$

Number of games played by men among themselves

$$= 2 \times {}^{x}C_2$$

$$= \cancel{2}\ \frac{x!}{(x-2)! \times \cancel{2}!} = x(x-1)$$

Number of games played by men with women

$$= 2(2x) = 4x$$

$x(x-1) - 4x = 66$

$x^2 - x - 4x = 66$

$x^2 - 5x - 66 = 0$

$x^2 - 11x + 6x - 66 = 0$

$(x-11)(x+6) = 0$

$x = 11, -6$

$11 \in [10, 12)$

ⓑ

2014 online

An eight- digit- number divisible by
is to be formed using digits fro
0 to 9 without repeating the dig
The number of ways in which th
can be done is

(a) 72(7!) (b) 18(7!) (c) 40(7!) (d) 3

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

There are 10 digits

- Add all the digits which is equal to = 45, divisible by

∴ For 8 digit- numbers we need to remove two digits from

we can only remove the follow pairs $(0, 9)$, $(1, 8)$, $(2, 7)$

$(3, 6)$, $(4, 5)$

because $0+9 = 9$, $45-9 = 36$ wh is divisible by 9

Similarly for other pairs

If (0,9) are removed Then There are 8 digits

possible number of 8 digits = 8!

If (1,8) are removed Then There are 8 digits 0, 2, 3, 4, 5, 6, 7, 9

Then possible number of 8 digits =

8! - 7!

[∵ subtracting the number of cases where 0 is at the left most place)

Similarly for other numbers i.e when we remove (2, 7) (3, 6) (4, 5) we get (8! - 7!) in each case.

Total number of 8 digit numbers

8! + (8! - 7!) + (8! - 7!) + (8! - 7!) + (8! - 7!)

8! + 4 (8! - 7!)

8! + 4.8! - 4.7!

5.8! - 4.7!

40.7! - 4.7! = 7! (40-4)

36.7! ⓓ

2014 online

The sum of the digits in the units place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6 without repetition is

(a) 432 (b) 108 (c) 36 (d) 18

_ _ _ 3

Fix 3 at the unit place. Then three places can be filled in 3! ways. Similarly fix 4, 5, 6 at unit place. Remaining places can be filled in 3! = 6 ways

we are to find the sum of the digits in the unit place. which is equal to

$$6(3+4+5+6)$$

$$6 \times 18 = 108$$

ⓑ

2015 offline

Let A and B be two sets containing 4 and 2 elements respectively. Then the number of subsets of the set A × B, each having at least three elements is

(a) 219 (b) 256 (c) 275 (d) 510

$n(A) = 4 \qquad n(B) = 2$

$n(A \times B) = n(A) \times n(B) = 4 \times 2 = 8$

Total number of subsets $= 2^8$

No of subsets of A × B which contain at least three elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 256 - (1 + 8 + 28)$

$= 256 - 37 = 219$

ⓐ

2015 online

If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is

(a) 10 (b) 12 (c) 9 (d) 16

We know if number of sides of a polygon is $n\ (n \geq 3)$ Then number of diagonals of the polygon are

$$^nC_2 - n$$

Here $^nC_2 - n = 54$

$$\frac{n!}{(n-2)!\,2!} - n = 54$$

$$\frac{n(n-1)}{2} - n = 54$$

$$n^2 - n - 2n = 108$$

$$n^2 - 3n - 108 = 0$$

$$(n-12)(n+9) = 0$$

$$n = 12 \qquad n = -9$$

Since $n \geq 3$ so $n = 12$

ⓑ

2015 offline

The number of integers greater than 6000 that can be formed using the digits 3, 5, 6, 7 and 8 without repetition is

(a) 216 (b) 192 (c) 120 (d) 72

The ~~digits~~ integer greater than 6000 may be of 4 digit number or 5 digit number.

when number is of 4 digit

| | | | |
|---|---|---|---|
| 3 | 4 | 3 | 2 |
| 1st | 2nd | 3rd | 4th |

First place can be filled in 3 ways.
Second place can be filled in 4 ways.
Third place can be filled in 3 ways
Fourth place can be filled in 2 ways
Total no of ways = 3 × 4 × 3 × 2
= 72 ways.

when number is of 5 digit
Total no of five digit number
$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$ ways.

Total no of integers greater than 6000 = 72 + 120 = 192 ⓑ