2011 off line

let $f : R \to [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists

and $\lim_{x \to 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$

Then $\lim_{x \to 5} f(x)$ equals

(a) 0 (b) 1 (c) 2 (d) 3

$$\lim_{x \to 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$$

$$\lim_{x \to 5} (f(x))^2 - 9 = 0$$

let $\lim_{x \to 5} f(x) = l$

$$l^2 - 9 = 0$$

$$l^2 = 9$$

$$l = \pm 3$$

$\lim_{x \to 5} f(x) = 3$ as $f(x) \geq 0$ $\forall x \in R$

(d)

2011 off-line

$$\lim_{x \to 2} \frac{\sqrt{1-\cos 2(x-2)}}{x-2}$$

(a) equals $\frac{1}{\sqrt{2}}$ (b) does not exist

(c) equals $\sqrt{2}$ (d) equals $-\sqrt{2}$

$$\lim_{x \to 2} \frac{\sqrt{2\sin^2(x-2)}}{x-2}$$

$$\lim_{x \to 2} \sqrt{2}\,\frac{\sin(x-2)}{x-2}$$

$$\lim_{x \to 2-} \sqrt{2}\,\frac{\sin(x-2)}{(x-2)}$$

$$\lim_{h \to 0} \sqrt{2}\,\frac{\sin(\cancel{2}-h-\cancel{2})}{(\cancel{2}-h-\cancel{2})}$$

$$\lim_{h \to 0} \sqrt{2}\,\frac{\sin(-h)}{-h} = \lim_{h \to 0} \sqrt{2}\,\frac{(-)\sin h}{h}$$

$$= -\sqrt{2}.$$

$$\lim_{x \to 2+} \sqrt{2}\,\frac{\sin(x-2)}{x-2}$$

$$\lim_{h \to 0} \sqrt{2}\,\frac{\sin(\cancel{2}+h-\cancel{2})}{\cancel{2}+h-\cancel{2}}$$

$$\lim_{h \to 0} \sqrt{2}\,\frac{\sin h}{h} = \sqrt{2}$$

$$\lim_{x \to 2+} \sqrt{2}\,\frac{\sin(x-2)}{x-2} \neq \lim_{x \to 2-} \sqrt{2}\,\frac{\sin(x-2)}{x-2}$$

Limit does not exist (b)

2011 offline

The values of p and q for which function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & x < 0 \\ q & x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & x > 0 \end{cases}$$

is continuous for all x in R are

(a) $p = \frac{1}{2}, q = \frac{3}{2}$ (b) $p = \frac{1}{2}, q = -\frac{3}{2}$

(c) $p = \frac{5}{2}, q = \frac{1}{2}$ (d) $p = -\frac{3}{2}, q = \frac{1}{2}$

$$\lim_{x \to 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$\lim_{x \to 0^-} \left(\frac{\sin(p+1)x}{x} + \frac{\sin x}{x} \right)$$

$$\lim_{h \to 0} \left(\frac{\sin(p+1)(-h)}{-h} + \frac{\sin(0-h)}{-h} \right)$$

$$\lim_{h \to 0} \left(\frac{\cancel{-}\sin(p+1)h}{\cancel{-}h} + \frac{\cancel{-}\sin h}{\cancel{-}h} \right)$$

$$\lim_{h \to 0} \left((p+1) \frac{\sin(p+1)h}{h(p+1)} + \frac{\sin h}{h} \right)$$

$$(p+1)+1 = p+2$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}$$

$$\lim_{x\to 0+} \frac{\sqrt{x(1+x)} - \sqrt{x}}{x^{3/2}}$$

$$\lim_{x\to 0+} \frac{\sqrt{x}\left(\sqrt{1+x} - 1\right)}{x^{3/2}}$$

$$\lim_{x\to 0+} \frac{\sqrt{x+1} - 1}{x^{\frac{3}{2}-\frac{1}{2}}}$$

$$\lim_{x\to 0+} \frac{\sqrt{x+1} - 1}{x} \times \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\lim_{x \to 0^+} \frac{x + \cancel{1} - \cancel{1}}{x(\sqrt{x+1}+1)}$$

$$\lim_{x \to 0^+} \frac{x}{x(\sqrt{x+1}+1)}$$

$$\lim_{h \to 0} \frac{0+h}{(0+h)(\sqrt{0+h+1}+1)}$$

$$\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{h+1}+1)}$$

$$\frac{1}{2}$$

For continuity $p+2 = \frac{1}{2} = q$

$$p = \frac{1}{2} - 2 = -\frac{3}{2}$$

$$q = \frac{1}{2}$$

(b)

2019 off line

Q $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$

(a) $a=1 \quad b=4$ (b) $a=1 \quad b=-4$

(c) $a=2 \quad b=-3$ (d) $a=2 \quad b=3$

Sol $\lim_{x\to\infty} \frac{(x^2+x+1) - (ax+b)(x+1)}{x+1} = 4$

$\lim_{x\to\infty} \frac{x^2+x+1 - ax^2 - ax + bx - b}{x+1} = 4$

$\lim_{x\to\infty} \frac{x^2(1-a) + x(1-a-b) + (1-b)}{x+1} = 4$

given is finite

$\Rightarrow$ deg of numerator and denominator is same

$1-a=0 \quad a=1$

$\lim_{x\to\infty} \frac{x^2(1-a) + x(1-1-b) + (1-b)}{x+1} = 4$

$\lim_{x\to\infty} \frac{x(-b) + 1 - b}{x+1} = 4$

$\lim_{x\to\infty} \frac{\cancel{x}\left[-b + \frac{1-b}{x}\right]}{\cancel{x}\left(1+\frac{1}{x}\right)} = 4$

$-b = 4 \qquad b = -4$

$a = 1 \quad b = -4$ (b)

2012 offline

If $f : R \to R$ is function defined by $f(x) = [x] \cos \frac{2x-1}{2} \pi$ whose $[x]$ denotes the greatest integer function. Then f is:

(a) discontinuous only at $x = 0$
(b) discontinuous only at non zero integral value of x
(c) continuous only at $x = 0$
(d) continuous for every real x.

$$f(x) = \begin{cases} -\cos\left(\frac{2x-1}{2}\right)\pi & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ \cos\left(\frac{2x-1}{2}\right)\pi & 1 \le x < 2 \\ 2\cos\left(\frac{2x-1}{2}\right)\pi & 2 \le x < 3 \end{cases}$$

It is continuous at all points except $\ldots -1, 0, 1, 2 \ldots$

Check continuity at $x = n \in I$

$$\lim_{x \to n^-} f(x) = \lim_{h \to 0} [n-h] \cos \frac{2(n-h)-1}{2} \pi$$

$$= (n-1) \cos \frac{2n-1}{2} \pi$$

$$= 0$$

$$\lim_{x \to n^+} f(x) = \lim_{h \to 0} [n+h] \cos \frac{2(n+h)-1}{2} \pi$$

$$= n \cos\left(\frac{2n-1}{2}\right)\pi$$

$$= 0$$

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$f(0) = \lim_{x \to 4-} f(x) = \lim_{x \to 4+} f(x)$

$\Rightarrow$ f is continuous at each $x \in R$

d

2013 offline

$\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is eq

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $-\frac{1}{}$

$\lim_{x\to 0} \frac{(2\sin^2 x)(3+\cos x)}{x\tan 4x}$

$\lim_{x\to 0} \cancel{2}\left(\frac{\sin^2 x}{x^2}\right)\cancel{x}\frac{(3+\cos x}{\left(\frac{\tan 4x}{4x}\right.}$

$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \frac{(3+\cos x)}{\left(\frac{\tan 4x}{4x}\right)}$

2

(c)

2014 online

Q. The function $f(x) = \begin{cases} \dfrac{\sqrt{2+\cos x} - 1}{(\pi - x)^2} & x \neq \pi \\ k & x = \pi \end{cases}$

is continuous at $x = \pi$ then k equals

(a) 2 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 0

f is continuous at $x = \pi$

$$f(\pi) = \lim_{x \to \pi} \frac{\sqrt{2+\cos x} - 1}{(\pi - x)^2}$$

Put $\theta = \pi - x$

$x = \pi - \theta$

$$f(\pi) = \lim_{\theta \to 0} \frac{\sqrt{2+\cos(\pi-\theta)} - 1}{(\pi - (\pi - \theta))^2}$$

$$k = \lim_{\theta \to 0} \frac{\sqrt{2-\cos\theta} - 1}{\theta^2}$$

$$k = \lim_{\theta \to 0} \frac{\sqrt{2-\cos\theta} - 1}{(\sqrt{2-\cos\theta} + 1)} \times \frac{(\sqrt{2-\cos\theta} + 1)}{\theta^2}$$

$$k = \lim_{\theta \to 0} \frac{(\sqrt{2-\cos\theta})^2 - 1}{\theta^2(\sqrt{2-\cos\theta} + 1)}$$

$$k = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2(\sqrt{2-\cos\theta} + 1)} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$k = \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta^2\left(\sqrt{2-\cos\theta}+1\right)(1+\cos\theta)}$$

$$k = \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta^2} \times \frac{1}{\left(\sqrt{2-\cos\theta}+1\right)(1+\cos\theta)}$$

$$k = 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(b)

2014 online

If $\lim_{x \to 2} \dfrac{\sin(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4} = 5$

Then k is equals to

(a) 0 (b) 1 (c) 2 (d) 3

$\lim_{x \to 2} \dfrac{\sin(x-2)\{x^2+(k-2)x-2k\}}{(x-2)^2} = 5$

$\lim_{x \to 2} \dfrac{\sin(x-2)\ (x+k)(x-2)}{(x-2)^2} = 5$

$\lim_{x \to 2} \dfrac{\sin(x-2)}{x-2} \times (x+k) = 5$

$2+k = 5$

$k = 3$

(d)

2024 online

Q. $f(x)$ is continuous and $f($

Then $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is

(a) $\frac{9}{2}$ (b) $\frac{2}{9}$ (c) 0 (d)

$$f \lim_{x\to 0}\left(\frac{1-\cos 3x}{x^2}\right)$$

$$f \lim_{x\to 0}\left(\frac{1-\cos 3x}{x^2} \times \frac{1+\cos}{1+\cos}\right.$$

$$f \lim_{x\to 0}\left(\frac{9}{9} \times \frac{\sin^2 3x}{x^2(1+\cos 3x}\right.$$

$$f \lim_{x\to 0}\left(9\left(\frac{\sin 3x}{3x}\right)^2 \times \frac{1}{1+}\right.$$

$$f\left(\frac{9}{2}\right)$$

which is equal to $\frac{2}{9}$

(b)

2014 offline

$\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

(a) $\frac{\pi}{2}$ (b) 1 (c) $-\pi$ (d) π

$\lim_{x \to 0} \frac{\sin(\pi(1-\sin^2 x))}{x^2}$

$\lim_{x \to 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$

$\lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{x^2}$

$\lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x} \times \frac{\pi \sin^2 x}{x^2}$

$\lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x} \times \frac{\sin^2 x}{x^2} \times \pi$

$\overline{\pi}$

(d)

2015 offline

Let k be a non zero real number If

$$f(x) = \begin{cases} \dfrac{(e^x - 1)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)} & x \neq 0 \\ 12 & x = 0 \end{cases}$$

is a continuous function, Then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

f is continuous for

$$12 = \lim_{x\to 0} \frac{(e^x-1)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}$$

$$12 = \lim_{x\to 0} \frac{\frac{(e^x-1)^2}{x^2} \times x^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}$$

$$12 = \lim_{x\to 0} \left(\frac{e^x-1}{x}\right)^2 \left(\frac{\frac{x}{k}\times k}{\sin\frac{x}{k}}\right) \frac{\frac{x}{4}\times 4}{\log\left(1+\frac{x}{4}\right)}$$

$$12 = \lim_{x\to 0} 4k\left(\frac{e^x-1}{x}\right)^2 \left(\frac{\frac{x}{k}}{\sin\frac{x}{k}}\right)\left(\frac{\frac{x}{4}}{\log\left(1+\frac{x}{4}\right)}\right)$$

$$12 = 4k(1)(1)$$

$$\therefore 3 = k \quad (c)$$

9015 off lin

Q. $\lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{\sin^2 x}$ is equal to

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) 2

$\lim_{x \to 0} \dfrac{e^{x^2} - 1 + 1 - \cos x}{\sin^2 x}$

$\lim_{x \to 0} \left(\dfrac{e^{x^2} - 1 / x^2}{\sin^2 x / x^2} + \dfrac{1 - \cos x}{\sin^2 x} \right)$

$\lim_{x \to 0} \dfrac{(e^{x^2} - 1) / x^2}{\sin^2 x / x^2} + \dfrac{\cancel{1 - \cos x}}{\cancel{(1 - \cos x)}(1 + \cos x)}$

$1 + \frac{1}{2} = \frac{3}{2}$

(b).

2015 offline

Q. $\lim_{x \to 0} \dfrac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

(a) 4 (b) 3 (c) 2 (d) $\frac{1}{2}$

$$\lim_{x \to 0} \left(\frac{1-\cos 2x}{x}\right)\left(\frac{3+\cos x}{\tan 4x}\right)$$

$$\lim_{x \to 0} \left(\frac{2\sin^2 x}{x}\right)\left(\frac{3+\cos x}{\tan 4x}\right)$$

$$\lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2 \times x \left(\frac{3+\cos x}{\frac{\tan 4x}{4x} \times 4x}\right)$$

$$\lim_{x \to 0} \cancel{2}\left(\frac{\sin x}{x}\right)^2 \times \cancel{x}\ \frac{3+\cos x}{\left(\frac{\tan 4x}{4x}\right) \times \cancel{4}\cancel{x}_{2}}$$

$$4 \times \frac{1}{2} = 2$$

(c)

2016 off line

. Let $p = \lim_{x \to 0+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ then

$\log p$ is equal to

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $= \frac{1}{4}$

$$p = \lim_{x \to 0+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$$

$$\log p = \log \lim_{x \to 0+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$$

$$= \lim_{x \to 0+} \log (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$$

$$= \lim_{x \to 0+} \frac{1}{2x} \log (1 + \tan^2 \sqrt{x})$$

$$= \frac{1}{2} \lim_{x \to 0+} \frac{\log (1 + \tan^2 \sqrt{x})}{(\sqrt{x})^2}$$

$$= \frac{1}{2} \lim_{x \to 0+} \frac{\log (1 + \tan^2 \sqrt{x})}{\tan^2 \sqrt{x}} \times$$

$$= \frac{1}{2} \lim_{x \to 0+} \frac{\log (1 + \tan^2 \sqrt{x})}{\tan^2 \sqrt{x}} \Big(\frac{\tan}{}$$

$$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

c

2016 online

If $\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$

Then a is equal to

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

$$\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$$

$$\lim_{x \to \infty} \left(1 + \frac{ax-4}{x^2}\right)^{2x}$$

$$\lim_{x \to \infty} \left[\left(1 + \frac{ax-4}{x^2}\right)^{\frac{x^2}{ax-4}}\right]^{\frac{ax-4}{x} \times 2} = e^3$$

$$\lim_{x \to \infty} (e)^{\left(a - \frac{4}{x}\right) \times 2} = e^3$$

$$\left[\because \lim_{x \to 0} (1+x)^{\frac{1}{x}} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e\right]$$

$$e^{2a} = e^3$$

$$2a = 3$$

$$a = \frac{3}{2}$$

(b)

2016 online

Q Let $a, b \in R$ $(a \neq 0)$ If the function f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & \sqrt{2} \leq x < \infty \end{cases}$$

is continuous in the interval $[a, \infty)$ then an ordered pair (a, b) is

(a) $(-\sqrt{2}, 1-\sqrt{3})$ (b) $(\sqrt{2}, -1+\sqrt{3})$

(c) $(\sqrt{2}, 1-\sqrt{3})$ (d) $(-\sqrt{2}, 1+\sqrt{3})$

Sol $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$$\lim_{x \to 1^-} \frac{2x^2}{a} = \lim_{x \to 1^+} a = a$$

$$\lim_{h \to 0} \frac{2(1-h)^2}{a} = a$$

$$\frac{2}{a} = a$$

$$a^2 = 2$$

$$a = \pm\sqrt{2}$$