2013 off line

Q If x, y, z are in AP and $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are also in AP then

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$ (d) $x = y = z$

Sol x, y, z are in AP

$$2y = x + z$$

$\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in AP

$$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$$

$$\tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$$\frac{2y}{1-y^2} = \frac{2y}{1-xz}$$

$$\frac{1}{1-y^2} = \frac{1}{1-xz}$$

$$\cancel{1} - xz = \cancel{1} - y^2$$

$$y^2 = xz$$

$$\left(\frac{x+z}{2}\right)^2 = xz$$

$$(x+z)^2 = 4xz$$

$$(x+z)^2 - 4xz = 0$$

$$(x-z)^2 = 0 \qquad x = z$$

$$\Rightarrow x = y = z \qquad (d)$$

Scanned with CamScanner

Q A value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$

(a) $-\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

Sol $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$

$\cot^{-1}(1+x) = \sin^{-1}(\cos(\tan^{-1}x$

$\cot^{-1}(1+x) = \sin^{-1}(\sin(\frac{\pi}{2}$

$\cot^{-1}(1+x) = \frac{\pi}{2} + \tan^{-1}x$

$(1+x) = \cot(\frac{\pi}{2} + \tan^{-1}$

$1+x = -\tan\tan^{-1}x$

$1+x = -x$

$2x = -1$

$x = -\frac{1}{2}$

a

CS Scanned with CamScanner

2013 online

Q The number of solution of the equation $\sin^{-1}x = 2\tan^{-1}x$ (in principal value) is

(a) 1 (b) 4 (c) 2 (d) 3

Sol $\sin^{-1}x = 2\tan x$

Put $x = \tan\theta$

$\sin^{-1}\tan\theta = 2\tan\tan^{-1}\theta$

$\sin^{-1}\tan\theta = 2\theta$

$\tan\theta = \sin 2\theta$

$\frac{\sin\theta}{\cos\theta} = 2\sin\theta\cos\theta$

$\sin\theta = 2\sin\theta\cos^2\theta$

$\sin\theta - 2\sin\theta\cos^2\theta = 0$

$\sin\theta(1-2\cos^2\theta) = 0$

either $\sin\theta = 0$ or $1-2\cos^2\theta = 0$

$1 = 2\cos^2\theta$

$\frac{1}{2} = \cos^2\theta$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ 3 value (d)

Scanned with CamScanner

2013 online

Q If $S = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \dots$

$\dots \tan^{-1}\frac{1}{1+(x+19)(x+20)}$ I

Then tan S is equal to

(a) $\frac{20}{401+20x}$ (b) $\frac{x}{x^2+20x+1}$

(c) $\frac{20}{x^2+20x+1}$ (d) $\frac{x}{401+20x}$

Sol I can be written as

$$\left(\tan^{-1}(x+1) - \tan^{-1}x\right) + \left(\tan^{-1}(x+2) - \tan^{-1}(x+1)\right) + \dots \left(\tan^{-1}(x+20) - \tan^{-1}(x+19)\right)$$

After adding we get -

$$S = \tan^{-1}(x+20) - \tan^{-1}x$$

$$S = \tan^{-1}\frac{\cancel{x}+20-\cancel{x}}{1+x(x+20)}$$

$$S = \tan^{-1}\frac{20}{1+x^2+20x}$$

$$\tan S = \frac{20}{x^2+20x+1}$$

(c)

Scanned with CamScanner

2013 off line

Q The value of $\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right\}$

(a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$ (d) $\frac{24}{23}$

Sol $\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+(2+4+6\cdots 2n)\right)\right\}$

$\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+(2+4+6\cdots 2n)\right)\right\}$

$\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+2(1+2+3\cdots n)\right)\right\}$

$\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+\cancel{2}\frac{n(n+1)}{\cancel{2}}\right)\right\}$

$\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+n(n+1)\right)\right\}$

$\cot\left\{\sum_{n=1}^{23}\tan^{-1}\frac{1}{1+n(n+1)}\right\}$

$\cot\left(\sum_{n=1}^{23}\tan^{-1}\frac{(n+1)-n}{1+n(n+1)}\right\}$

$\cot\left(\sum_{n=1}^{23}\tan^{-1}(n+1)-\tan^{-1}(n)\right\}$

$\cot\left[\left(\tan^{-1}(2)-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)\cdots\right.$
$\left.\cdots\left(\tan^{-1}24-\tan^{-1}23\right)\right]$

CS Scanned with CamScanner

$$\cot\left(\tan^{-1}24 - \tan^{-1}1\right)$$

$$\cot\left(\tan^{-1}\frac{24-1}{1+24\times 1}\right)$$

$$\cot\left(\tan^{-1}\frac{23}{25}\right.$$

$$\cot\ \cot^{-1}\frac{25}{23}$$

$$\frac{25}{23}.$$

CS Scanned with CamScanner

2014 online

Q Statement-I: The equation

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$ For any $x \in R$

Statement II: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ and

$0 \leq \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$

(a) Both statement I and II are true
(b) Both statement I and II are false
(c) Statement I is true and statement II is false
(d) Statement I is false and statement II is True

Sol $\sin^{-1}x \leq \frac{\pi}{2}$

$(\sin^{-1}x)^3 \leq \left(\frac{\pi}{2}\right)^3 = \frac{\pi^3}{8}$

$\cos^{-1}x \leq \pi$

$(\cos^{-1}x)^3 \leq \pi^3$

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 \leq \frac{\pi^3}{8} + \pi^3$

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 \leq \frac{9}{8}\pi^3$

$\Rightarrow (\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3 \geq \frac{1}{32}\pi^3$

has no solution

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for $-1 \leq x \leq 1$

not for each real x

So both statements are wrong (b)

Scanned with CamScanner

2014 online.

Q The principal value of $\tan^{-1}\left(\cot \frac{43\pi}{4}\right)$ is

(a) $-\left(\frac{3\pi}{4}\right)$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $-\left(\frac{\pi}{4}\right)$

Sol $\tan^{-1}\left(\cot \frac{43\pi}{4}\right)$

$\tan^{-1}\left(\cot\left(11\pi - \frac{\pi}{4}\right)\right)$

$\tan^{-1}\left(-\cot \frac{\pi}{4}\right)$

$\tan^{-1}(-1)$

$= -\frac{\pi}{4}$ (d)

CS Scanned with CamScanner

2015 online

Q Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

where $|x| < \frac{1}{\sqrt{3}}$

Then a value of y is

(a) $\frac{3x-x^3}{1-3x^2}$ (b) $\frac{3x+x^3}{1-3x^2}$ (c) $\frac{3x-x^3}{1+3x^2}$

(d) $\frac{3x+x^3}{1+3x^2}$

Sol $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1} x + 2\tan^{-1} x$

$= 3\tan^{-1} x$

$\tan^{-1} y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$y = \frac{3x-x^3}{1-3x^2}$

(a)

Scanned with CamScanner

2015 archive

Q 91. $f(x) = 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ (I)

Then $f(5)$ is equal to

(a) $\frac{\pi}{2}$ (b) π (c) $4\tan^{-1}5$ (d)

Sol

We know $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan$

I becomes

$f(x) = 2\tan^{-1} + (\pi - 2\tan$

$= \cancel{2\tan^{-1}x} + \pi - \cancel{2\tan^{-1}}$

$= \pi$

$f(x) = \pi$

$f(5) = \pi$

(b)

Scanned with CamScanner