2012 offline

Q. If the integral $\int \frac{5\tan x}{\tan x - 2}\,dx = x + a\log|\sin x - 2\cos x| + k$

Then a is equal to

(a) -2 (b) 1 (c) 2 (d) -1

Sol $\int \frac{5\tan x}{\tan x - 2}\,dx.$

$$\int \frac{5\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2}\,dx = \underbrace{\int \frac{5\sin x}{\sin x - 2\cos x}\,dx}_{I}$$

Let $5\sin x = A(\sin x - 2\cos x) + B\frac{d}{dx}(\sin x - 2\cos x)$

$= A(\sin x - 2\cos x) + B(\cos x + 2\sin x)$

$= A\sin x - 2A\cos x + B\cos x + 2B\sin x$

$= (A + 2B)\sin x - (2A - B)\cos x.$

Comparing the co-eff. of $\sin x$ and $\cos x$

$5 = A + 2B \qquad 0 = -2A + B$

After solving we get $A = 1$ $B = 2$

$5\sin x$ becomes $(\sin x - 2\cos x) + 2(\cos x + 2\sin x)$

I become $\int \frac{\sin x - 2\cos x + 2(\cos x + 2\sin x)}{\sin x - 2\cos x}\,dx$

$$\int \left(\frac{\sin x - 2\cos x}{\sin x - 2\cos x} + \frac{2(\cos x + 2\sin x)}{\sin x - 2\cos x} \right) dx$$

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2012 offline

$$\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}\,dx$$ equals to

for some arbitrary constant. k

(a)

(a) $\frac{-1}{(\sec x+\tan x)^{1/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+k$

(b) $\frac{+1}{(\sec x+\tan x)^{\frac{1}{2}}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+k$

(c) $\frac{-1}{(\sec x+\tan x)^{\frac{1}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+k$

(d) $\frac{1}{(\sec x+\tan x)^{\frac{1}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+k$

$$\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}\,dx \qquad I$$

$\sec x+\tan x = t$

$(\sec x\tan x+\sec^2 x)\,dx = dt$

$\sec x(\tan x+\sec x)\,dx = dt$

$\sec x \cdot t\,dx = dt$

$\sec x\,dx = \frac{1}{t}\,dt$

We know $\sec^2 x-\tan^2 x = 1$

$(\sec x-\tan x)(\sec x+\tan x) = 1$

$(\sec x-\tan x)t = 1$

$\sec x-\tan x = \frac{1}{t}$

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$\sec x + \tan x = t$

$\sec x - \tan x = \frac{1}{t}$

$2\sec x = t + \frac{1}{t}$

$\sec x = \frac{1}{2}\left(t + \frac{1}{t}\right)$

I becomes

$$\int \frac{\frac{1}{2}\left(t + \frac{1}{t}\right) \frac{1}{t}\, dt}{t^{9/2}}$$

$$\frac{1}{2}\int \frac{\left(1 + \frac{1}{t^2}\right)}{t^{9/2}}\, dt$$

$$\frac{1}{2}\int \left(\frac{1}{t^{9/2}} + \frac{1}{t^{13/2}}\right) dt$$

$$-\frac{1}{2}\left\{\frac{2}{7t^{\frac{7}{2}}} + \frac{2}{11t^{\frac{11}{2}}}\right) + k$$

$$-\frac{1}{7t^{\frac{7}{2}}} - \frac{1}{11t^{\frac{11}{2}}} + k$$

$$-\left[\frac{1}{7(\sec x + \tan x)^{\frac{7}{2}}} + \frac{1}{11(\sec x + \tan x)^{\frac{11}{2}}}\right] + k$$

$$-\frac{1}{(\sec x + \tan x)^{11/2}}\left[\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + k$$

(c)

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2013 online

Q. $\int \frac{x^2-x+1}{x^2+1} e^{\cot^{-1}x}\, dx = A(x)\, e^{\cot^{-1}x} + c$

(a) $-x$ (b) x (c) $\sqrt{1-x}$ (d) $\sqrt{1+x}$

Sol- $\int \frac{x^2-x+1}{x^2+1} e^{\cot^{-1}x}\, dx$

$$x = \cot t$$
$$dx = -\operatorname{cosec}^2 t\, dt$$

$$\int \frac{\cot^2 t - \cot t + 1}{\cot^2 t + 1} e^t (-\operatorname{cosec}^2 t)\, dt$$

$$-\int \frac{\cot^2 t - \cot t + 1}{\cancel{\operatorname{cosec}^2 t}} e^t (\cancel{\operatorname{cosec}^2 t})\, dt$$

$$\int (\cot t - \cot^2 t - 1) e^t\, dt$$

$$\int e^t (\cot t - (\cot^2 t + 1))\, dt$$

$$\int e^t (\cot t - \operatorname{cosec}^2 t)\, dt$$

$$e^t \cot t + c$$

$$x\, e^{\cot^{-1}x} + c = A(x)\, e^{\cot^{-1}x} + c$$

$$A(x) = x$$

(b)

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2013 online

The integral $\int \frac{x\,dx}{2-x^2+\sqrt{2-x^2}}$ equals

(a) $\log \cancel{x} |1+\sqrt{2+x^2}| + c$

(b) $-\log |1+\sqrt{2-x^2}| + c$

(c) $-x \log |1-\sqrt{2-x^2}| + c$

(d) $x \log |1-\sqrt{2+x^2}| + c$

$$\int \frac{x \quad dx}{2-x^2+\sqrt{2-x^2}}$$

$$\int \frac{x}{\sqrt{2-x^2}\left(\sqrt{2-x^2}+1\right)}\,dx$$

$$\sqrt{2-x^2} = t$$

$$\frac{1}{\cancel{2}\sqrt{2-x^2}}(-\cancel{2}x)\,dx = dt$$

$$\frac{-x\,dx}{\sqrt{2-x^2}} = dt$$

$$-\int \frac{dt}{(t+1)}$$

$$-\log|1+t| + c$$

$$-\log|1+\sqrt{2-x^2}| + c$$

(b)

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2013 online

Q. If $\int \frac{dx}{x+x^7} = p(x)$ Then $\int \frac{x^6}{x+x^7}$

(a) $\log|x| - p(x) + C$

(b) $\log|x| + p(x) + C$

(c) $x - p(x) + C$

(d) $x + p(x) + C$

$$\int \frac{x^6}{x+x^7}\,dx$$

$$\int \frac{x^6}{x(1+x^6)}\,dx$$

$$\int \frac{x^6+1-1}{x(1+x^6)}\,dx$$

$$\int \frac{\cancel{x^6+1}}{x(\cancel{1+x^6})}\,dx - \int \frac{1}{x(1+x^6)}\,d$$

$$\log|x| - p(x) + C$$

(a)

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2013 off line

If $\int f(x)\,dx = \psi(x)$ then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x)^3 - 3\int x^3\psi(x^3)\,dx + c$

(b) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + c$

(c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)\,dx\right] + c$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^2)\,dx\right] + c$

$$I = \int x^5 f(x^3)\,dx$$

Put $x^3 = t$

$$3x^2\,dx = dt$$

$$x^2\,dx = \frac{1}{3}dt$$

$$\int x^3 \cdot x^2 f(x^3)\,dx$$

$$\frac{1}{3}\int t\,f(t)\,dt$$

$$\frac{1}{3}\left[t\int f(t)\,dt - \int \int f(t)\,dt\right]$$

$$\frac{1}{3}\left[t\,\psi(t) - \int \psi(t)\,dt\right]$$

$$\frac{1}{3}\left[x^3\psi(x^3) - \int \psi(x^3)\,3x^2\,dx\right]$$

$$\frac{1}{3}x^3\psi(x^3) - \int \psi(x^3)\,x^2\,dx + c$$

(b)

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goru off line

Q. The integral $\int \left(1+x-\frac{1}{x}\right) e^{x+\frac{1}{x}} dx$

(a) $(x-1) e^{x+\frac{1}{x}} + c$ (b) $x e^{x+\frac{1}{x}} + c$

(c) $(x+1) e^{x+\frac{1}{x}} + c$ (d) $-x e^{x+\frac{1}{x}}$

Sol

$$\int e^{x+\frac{1}{x}} \left(1+x-\frac{1}{x}\right) dx$$

$$\int e^{x+\frac{1}{x}} \left(1+x\left(1-\frac{1}{x^2}\right)\right) dx$$

$$\int \left(e^{x+\frac{1}{x}} + x\left(1-\frac{1}{x^2}\right) e^{x+\frac{1}{x}}\right)$$

$$\int \frac{d}{dx}\left(\left(e^{x+\frac{1}{x}}\right) x\right) dx$$

$$x e^{x+\frac{1}{x}} + C$$

b

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Q. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx$ is equal to

(a) $\frac{1}{2}\sin 2x + c$ (b) $-\frac{1}{2}\sin 2x + c$

(c) $-\frac{1}{2}\sin x + c$ (d) $-\sin^2 x + c$

$$\sin^8 x - \cos^8 x = (\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)$$

$$(1 - 2\sin^2 x \cos^2 x)(\sin^2 x - \cos^2 x)$$

$$-(1 - 2\sin^2 x \cos^2 x)(\cos 2x)$$

$$-\int \frac{\cancel{(1 - 2\sin^2 x \cos^2 x)}(\cos 2x)}{\cancel{1 - 2\sin^2 x \cos^2 x}}\, dx$$

$$-\int \cos 2x \, dx$$

$$-\frac{\sin 2x}{2} + c$$

$$-\frac{1}{2}\sin 2x + c$$

(b)

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The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ is equal to

(a) $\frac{1}{1+\cot^3 x} + c$ (b) $-\frac{1}{3(1+\tan^3 x)} + c$

(c) $\frac{\sin^3 x}{1+\cos^3 x} + c$ (d) $\frac{\cos^3 x}{3(1+\sin^3 x)} + c$

Divide num. and den. by $\cos^6 x$.

$$\int \frac{\frac{\sin^2 x \cos^2 x}{\cos^6 x}}{\left(\frac{\sin^3 x + \cos^3 x}{\cos^3 x}\right)^2} dx$$

$$\int \frac{\tan^2 x \sec^2 x}{(\tan^3 x + 1)^2} dx$$

$$1+\tan^3 x = t$$

$$3\tan^2 x \sec^2 x \, dx = dt$$

$$\frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t} + c$$

$$-\frac{1}{3t} + c$$

$$-\frac{1}{3(1+\tan^3 x)} + c.$$

(b)

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2014 online

If m is non zero number and

$$\int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^{m} + 1)^3}\, dx = f(x) + c$$

Then $f(x)$ is

(a) $\dfrac{x^{5m}}{2m(x^{2m}+x^{m}+1)^2}$ (b) $\dfrac{x^{4m}}{2m(x^{2m}+x^{m}+1)^2}$

(c) $\dfrac{2m(x^{5m}+x^{4m})}{(x^{2m}+x^{m}+1)^2}$ (d) $\dfrac{x^{5m}+x^{4m}}{2m(x^{2m}+x^{m}+1)^2}$

$$\int \frac{x^{5m-1} + 2x^{4m-1}}{x^{6m}(1 + x^{-m} + x^{-2m})^3}\, dx$$

$$\int \frac{x^{-6m}(x^{5m-1} + 2x^{4m-1})}{(1 + x^{-m} + x^{-2m})^3}\, dx$$

$$\int \frac{x^{-m-1} + 2x^{-2m-1}}{(1 + x^{-m} + x^{-2m})^3}\, dx$$

$$1 + x^{-m} + x^{-2m} = t$$

$$(-m x^{-m-1} - 2m x^{-2m-1})\, dx = dt$$

$$-m(x^{-m-1} + 2x^{-2m-1})\, dx = dt$$

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$$= -\frac{1}{m}\int \frac{1}{t^3}\,dt$$

$$= \frac{1}{2mt^2}$$

$$= \frac{1}{2m(1+x^{-m}+x^{-2m})^2}$$

$$= \frac{1}{2m\left(1+\frac{1}{x^m}+\frac{1}{x^{2m}}\right)^2}$$

$$= \frac{1}{2m\left(\frac{x^{2m}+x^m+1}{x^{2m}}\right)^2}$$

$$= \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$$

(b)

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2015 off line

The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals

(a) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (b) $(x^4+1)^{\frac{1}{4}} + c$

(c) $-(x^4+1)^{\frac{1}{4}} + c$ (d) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

$$\int \frac{dx}{x^2(x^4)^{3/4}(1+x^{-4})^{3/4}}$$

$$\int \frac{dx}{x^2\, x^3 (1+x^{-4})^{3/4}}$$

$$1 + x^{-4} = t$$

$$-4x^{-5}\,dx = dt$$

$$x^{-5}\,dx = -\frac{1}{4}\,dt$$

$$-\frac{1}{4}\int \frac{dt}{t^{3/4}} = -\frac{1}{4} \times 4\, t^{\frac{1}{4}} + c$$

$$= -t^{\frac{1}{4}} + c$$

$$= -\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + c$$

$$= -\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$$

d

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The integral $I = \int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

(a) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + c$ (b) $4\left(\frac{x-2}{x+2}\right)^{\frac{1}{4}} + c$

(c) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + c$ (d) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}} + c$

$$I = \int \frac{dx}{(x+1)^{3/4}(x-2)^{\frac{5}{4}}}$$

$$\int \frac{dx}{(x+1)^{2-\frac{5}{4}}(x-2)^{5/4}}$$

$$\int \frac{dx}{\frac{(x+1)^2}{(x+1)^{5/4}}(x-2)^{5/4}}$$

$$\int \frac{dx}{(x+1)^2\left(\frac{x-2}{x+1}\right)^{5/4}}$$

$$\frac{x-2}{x+1} = t \qquad \frac{1}{(x+1)^2}dx = \frac{1}{3}dt$$

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$-\frac{1}{3}\int \frac{1}{t^{5/4}}\, dt =$

$-\frac{1}{3}\, 4\, t^{-\frac{1}{4}} + C$

$= -\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{-\frac{1}{4}} + C$

$= -\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$

(c)

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2015 online

If $I = \int \frac{\log(t + (1+t^2))}{\sqrt{1+t^2}} dt = \frac{1}{2}(g(t))^2 + c$ where

c is a constant. Then $(g(2))$ is equal to

(a) $2\log(2+\sqrt{5})$ (b) $\log(2+\sqrt{5})$

(c) $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$ (d) $\frac{1}{2}\log(2+\sqrt{5})$

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Let $\log\left(t+\sqrt{1+t^2}\right) = u$

$$\frac{1}{t+\sqrt{1+t^2}}\left(1+\frac{t}{\sqrt{1+t^2}}\right)dt = du$$

$$\frac{1}{\cancel{\left(t+\sqrt{1+t^2}\right)}}\left(\frac{\cancel{\sqrt{1+t^2}+t}}{\sqrt{1+t^2}}\right)dt = du$$

$$\frac{1}{\sqrt{1+t^2}}\,dt = du$$

$$I = \int u\,du = \frac{1}{2}\left(g(t)\right)^2$$

$$\frac{u^2}{2} + \cancel{c} = \frac{1}{2}\left(g(t)\right)^2 + \cancel{c}$$

$$u^2 = \left(g(t)\right)^2$$

$$u = g(t)$$

$$\log\left(t+\sqrt{1+t^2}\right) = g(t)$$

Put $t = 2$

$$\log\left(2+\sqrt{1+4}\right) = g(2)$$

$$\log\left(2+\sqrt{5}\right) = g(2)$$

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(b)

2016 off line

Q. The integral $I = \int \dfrac{2x^{12} + 5x^{9}}{(x^5 + x^3 + 1)^3}\, dx$ is equal to

(a) $\dfrac{-x^5}{(x^5 + x^3 + 1)^2} + c$ (b) $\dfrac{x^{10}}{2(x^5 + x^3 + 1)^5} + c$

(c) $\dfrac{x^5}{2(x^5 + x^3 + 1)} + c$ (d) $\dfrac{-x^{10}}{2(x^5 + x^2 + 1)^2} + c$

Sol

$$\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}\, dx$$

$$\int \frac{2x^{12} + 5x^9}{x^{15}(1 + x^{-2} + x^{-5})^3}\, dx$$

$$\int \frac{x^{-15}(2x^{12} + 5x^9)}{(1 + x^{-2} + x^{-5})^3}\, dx$$

$$\int \frac{2x^{-3} + 5x^{-6}}{(1 + x^{-2} + x^{-5})^3}\, dx$$

$$1 + x^{-2} + x^{-5} = t$$

$$\left(-2x^{-3} + (-5)x^{-6}\right) dx = dt$$

$$-\left(2x^{-3} + 5x^{-6}\right) dx = dt$$

$$-\int \frac{dt}{t^3} = \frac{1}{2t^2} + c$$

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$$\frac{1}{2\left(1+x^{-2}+x^{-5}\right)^2}+C$$

$$\frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$$

$$\frac{1}{2\left(\frac{x^5+x^3+1}{x^5}\right)^2}+C$$

$$\frac{1}{2}\frac{x^{10}}{(x^5+x^3+1)^2}+C$$

$$\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

b

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2016 online

Q The integral $I = \int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to where c is a constant of integration

(a) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$ (b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + c$

Sol $I \quad \int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$

$$\int \frac{dx}{(1+\sqrt{x})\sqrt{x}(\sqrt{1-x}}$$

$$\sqrt{x} = t \qquad \frac{1}{2\sqrt{x}}\,dx = dt$$

$$\int \frac{2\,dt}{(1+t)\sqrt{1-t^2}}$$

$$1+t = \frac{1}{u}$$

$$dt = -\frac{1}{u^2}\,du$$

$$\int \frac{-\frac{1}{u^2}du}{\frac{1}{u}\sqrt{1-\left(\frac{1}{u}-1\right)^2}}$$

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