2011 offline

Q. 21. $\frac{dy}{dx} = y+3 > 0$ and $y(0) = 2$

$y(\log 2)$ is equal to

(a) -2 (b) 7 (c) 5 (d) 13

Sol $\frac{dy}{dx} = y+3$ $\qquad \frac{dy}{y+3} = dx$

$$\int \frac{1}{y+3}\,dy = \int 1 \cdot dx$$

$$\log(y+3) = x + c$$

$x = 0 \quad y = 2$

$$\log 5 = c$$

$$\log(y+3) = x + \log 5$$

Find $y(\log 2) = ?$

$$x = \log 2$$

$$\log(y+3) = \log 2 + \log 5$$

$$\log(y+3) = \log 10$$

$$y+3 = 10$$

$$y = 7 \quad (b)$$

2011 offline

Q. Consider the differential equation

$$y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$$

If $y(1) = 1$ Then x is given by

(a) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (b) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{y}$

(c) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (d) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{y}$

Sol

$$y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$$

$$y^2 dx + x dy - \frac{1}{y} dy = 0$$

Divide by $y^2 dy$.

$$\frac{dx}{dy} - \frac{x}{y^2} = \frac{1}{y^3} \quad \text{I}$$

$$IF = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

Multiply I by $e^{-\frac{1}{y}}$

$$\frac{d}{dx}\left(x e^{-\frac{1}{y}}\right) = \frac{1}{y^3} e^{-\frac{1}{y}}$$

integrate

$$x e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

I

$$I = \int \frac{1}{y^2} e^{-\frac{1}{y}} dy$$

Let $-\frac{1}{y} = t$ $\quad \frac{1}{y^2} dy = dt$

$$\int \underset{I}{-t} \; \underset{II}{e^t} \, dt$$

$$-te^t + \int e^t dt$$

$$-te^t + e^t$$

$$e^t(-t+1)$$

$$e^t(1-t) = e^{-\frac{1}{y}}\left(1+\frac{1}{y}\right)$$

Substitute in I

$$x e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(1+\frac{1}{y}\right) + c$$

$$x = \left(1+\frac{1}{y}\right) + ce^{+\frac{1}{y}}$$

When $x = 1$ $\quad y = 1$

$$1 = (1+1) + ce$$

$$-\frac{1}{e} = C$$

$$x = 1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$$

2012 offline

Q. The population $p(t)$ at time t of certain mouse species satisfies the differential equation $\frac{d}{dt} p(t) = 0.5\, p(t) - 450$

If $p(0) = 850$ then the time at which the population becomes zero is

(a) $\log 9$ (b) $\frac{1}{9}\log 18$ (c) $\log 18$ (d) $2\log 18$

Sol

$$\frac{d}{dt} p(t) - 0.5\, p(t) = -450 \quad \text{I}$$

$$\text{I.F} = e^{-\int 0.5\,dt} = e^{-0.5t} = e^{-0.5t}$$

Multiply I by $-0.5t$

$$\frac{d}{dt}\left(p(t)\, e^{-0.5t} \right) = -450 \left(e^{-0.5t} \right)$$

Integrati

$$p(t)\, e^{-0.5t} = +450 \times 2\, e^{-0.5t} + C$$

Put $t = 0$

$$p(0)\, e^{0} = 900 + C$$

$$850 = 900 + C$$

$$C = -50$$

$$p(t) = 900\, e^{-0.5t} + C$$

$$p(t) = 900\, e^{0.5t} - 50$$

$$p(t) = 0$$

$$0 = 900\, e^{-0.5t} - 50$$

$$\frac{50}{900} = e^{-0.5t}$$

18

$$\frac{1}{18} = e^{-0.5t}$$

$$e^{-0.5t} = \frac{1}{18}$$

$$-0.5t = \log \frac{1}{18}$$

$$-0.5t = \log 1 - \log 18$$

$$\cancel{-}0.5t = \cancel{-}\log 18$$

$$t = 2\log 18$$

d

2013 offline

Q At present a firm is manufacturing 9000 items. It is estimated that the change of production P w.r.t additional number of workers is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$

If the firm employs 25 more workers then the new level of production of items is

(a) 3000 (b) 3500 (c) 4500 (d) 2500

Sol

$$\frac{dp}{dx} = 100 - 12\sqrt{x}$$

$$dp = (100 - 12\sqrt{x})\,dx$$

$$\int dp = \int (100 - 12\sqrt{x})\,dx$$

$$p(x) = 100x - 8x^{\frac{3}{2}} + C$$

$$P(0) = 2000$$

$$2000 = C$$

$$p(25) = 100(25) - 8(25)^{3/2} + 2000$$

$$= 2500 - 8 \times 125 + 2000$$

$$= 2500 - 1000 + 2000$$

$$= 3500$$

(b)

2014 online

Q If the differential equation representing the family of all circles touching x-axis at the origin is

$$(x^2-y^2)\frac{dy}{dx} = g(x)\,y \text{ then } g(x) \text{ equal}$$

(a) $\frac{1}{2}x$ (b) $2x^2$ (c) $2x$ (d) $\frac{1}{2}x^2$

Sol An equation of circle touching the x-axis at the origin

$$x^2+(y-a)^2 = a^2 \quad \text{I}$$

where a is a parameter

Differentiate w.r.t x

$$2x + 2(y-a)y' = 0$$

$$2x + 2yy' - 2ay' = 0$$

$$2x + 2yy' = 2ay'$$

$$x + yy' = ay'$$

$$\frac{x}{y'} + y = a$$

Put this value in I

$$x^2 + \left(y - \left(\frac{x}{y'} + y\right)\right)^2 = \left(\frac{x}{y'} + y\right)^2$$

$$x^2 + \left(\cancel{y} - \frac{x}{y'} - \cancel{y}\right)^2 = \left(y + \frac{x}{y'}\right)^2$$

$$x^2 + \frac{x^2}{y'^2} = \left(y + \frac{x}{y'}\right)^2$$

$$x^2 + \frac{\cancel{x^2}}{\cancel{y'^2}} = y^2 + \frac{\cancel{x^2}}{\cancel{y'^2}} + \frac{2xy}{y'}$$

$$x^2 = y^2 + \frac{2xy}{y'}$$

$$x^2 - y^2 = (2xy)\frac{1}{y'}$$

$$(x^2 - y^2)\frac{dy}{dx} = 2xy$$

$$(x^2 - y^2)\frac{dy}{dx} = (2x)(y)$$

$$\therefore \quad g(x) = 2x.$$

2014 Online

Q If the general solution of the differential equation

$$y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right) \text{ for some}$$

function ϕ is given by

$y \log|cx| = x$ where c is an arbitrary constant. Then $\phi(2)$ is equal to

(a) 4 (b) $\frac{1}{4}$ (c) -4 (d) $-\frac{1}{4}$

Sol
$$y \log|cx| = x$$

$$y' \log|cx| + y\frac{1}{\cancel{c}x}\cancel{c} = 1$$

$$y' \log|cx| + \frac{y}{x} = 1$$

$$y' \frac{x}{y} + \frac{y}{x} = 1$$

$y \log|cx| = x$

$\log|cx| = \frac{x}{y}$

$$y'\left(\frac{x}{y}\right) = 1 - \frac{y}{x}$$

$$y' = \frac{y}{x}\left(1 - \frac{y}{x}\right)$$

$$y' = \frac{y}{x} - \left(\frac{y}{x}\right)^2$$

Substitute in I.

$$\cancel{\frac{y}{x}} - \left(\frac{y}{x}\right)^2 = \cancel{\frac{y}{x}} + \phi\left(\frac{x}{y}\right)$$

$$-\left(\frac{y}{x}\right)^2 = \phi\left(\frac{x}{y}\right)$$

Find $\phi(2) \Rightarrow \frac{x}{y} = 2$

$x = 2y$

$$-\left(\frac{\cancel{y}}{2\cancel{y}}\right)^2 = \phi(2)$$

$$-\left(\frac{1}{2}\right)^2 = \phi(2)$$

$$-\frac{1}{4} = \phi(2)$$

d

2014 online

Q The general solution of the differential equation

$$\sin 2x \left(\frac{dy}{dx} - \sqrt{\tan x} \right) - y = 0 \text{ is}$$

(a) $y\sqrt{\tan x} = x + c$

(b) $y\sqrt{\cot x} = \tan x + c$

(c) $y\sqrt{\tan x} = \cot x + c$

(d) $y\sqrt{\cot x} = x + c$

Sol $\dfrac{dy}{dx} - \sqrt{\tan x} = \dfrac{y}{\sin 2x}$.

$$\frac{dy}{dx} - \frac{y}{\sin 2x} = \sqrt{\tan x} \quad \text{I}$$

$$\frac{dy}{dx} - \operatorname{cosec} 2x \, y = \sqrt{\tan x}.$$

$$\text{IF} = e^{-\int \operatorname{cosec} 2x \, dx}$$

$$= e^{-\frac{1}{2} \log \tan x}$$

$$= e^{\log (\tan x)^{-\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{\tan x}}$$

multiply I by $\frac{1}{\sqrt{\tan x}}$

$$\frac{d}{dx}\left(\frac{y}{\sqrt{\tan x}}\right) = 1$$

$$y\sqrt{\cot x} = x + c$$ d

2014 online

Q 21. $\frac{dy}{dx} + y\tan x = \sin 2x$ and
Then $y(\pi)$ is equal to
(a) 1 (b) -1 (c) -5 (d)

Sol
$$\frac{dy}{dx} + y\tan x = \sin 2x$$
$$I.F = e^{\int \tan x\, dx} = e^{\log \sec}$$
Multiply I by $\sec x$
$$\frac{d}{dx}(y\sec x) = (\sin 2x)(\sec$$
$$\int \frac{d}{dx}(y\sec x) = \int 2\sin x \cancel{\cos x}$$
$$y\sec x = -2\cos x +$$
Put $x = 0$, $y = 1$
we get $c = 3$
when $x = \pi$ then $y =$

2014 offline

Q Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{d}{dt}p(t) = \frac{1}{2}p(t) - 200$

If $p(0) = 100$ Then $p(1)$ equals

(a) $400 - 300\, e^{\frac{t}{2}}$

(b) $300 - 200\, e^{-\frac{t}{2}}$

(c) $600 - 500\, e^{t/2}$

(d) $400 - 300\, e^{-t/2}$

Sol
$$\frac{d}{dt}p(t) = \frac{1}{2}p(t) - 200$$

$$\frac{d}{dt}p(t) - \frac{1}{2}p(t) = -200$$

$$I\,F\;\; e^{-\int \frac{1}{2}dt} = e^{-\frac{1}{2}t}$$

Multiply both side by $e^{-\frac{t}{2}}$

$$\frac{d}{dt}\left(e^{-\frac{t}{2}}p(t)\right) = -200\, e^{-\frac{t}{2}}$$

Integrati

$$e^{-\frac{1}{2}t} p(t) = 400 e^{-\frac{t}{2}} + c$$

Put $t = 0$

$$100 = 400 + c$$

$$c = -300$$

$$p(t) = 400 e^{-\frac{t}{2}} + c$$

$$= 400 e^{-\frac{t}{2}} - 300$$

$$p(t) = 400 - 300 e^{\frac{t}{2}}$$ (a)

2015 online

Q The solution of the differential equation $y\,dx - (x+2y^2)\,dy = 0$ is $x = f(y)$.
If $f(-1) = 1$ then $f(1)$ is equal to

(a) 4 (b) 3 (c) 2 (d) 1

Sol
$$y\,dx - (x+2y^2)\,dy = 0$$
$$y\,dx - x\,dy - 2y^2\,dy = 0$$
Divide by $y\,dy$
$$\frac{\cancel{y}\,dx}{\cancel{y}\,dy} - \frac{x\,\cancel{dy}}{y\,\cancel{dy}} - \frac{2y^{\cancel{2}}\,\cancel{dy}}{\cancel{y}\,\cancel{dy}} = 0$$
$$\frac{dx}{dy} - x\cdot\frac{1}{y} = 2y \qquad \text{I}$$
$$IF = e^{-\int \frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$$
Multiply I by $\frac{1}{y}$
$$\frac{1}{y}\frac{dx}{dy} + \left(-\frac{1}{y^2}\right) = 2$$
$$\frac{d}{dy}\left(x\cdot\frac{1}{y}\right) = 2$$
$$\int \frac{d}{dy}\left(\frac{x}{y}\right) = \int 2\,dy$$
$$\frac{x}{y} = 2y + C$$

$$x = 2y^2 + cy$$

when $y = -1$, $x = 1$

$$1 = 2(-1)^2 + c(-1)$$

$$1 = 2 - c$$

$$c = 1$$

$$x = 2y^2 + (1)y$$

$$x = 2y^2 + y$$

$$f(y) = 2y^2 + y$$

$$f(1) = 2(1)^2 + 1 = 2 + 1$$

$$f(1) = 3 \quad b$$

2015 online

Q. $y(x)$ is the solution of the differential equation

$$(x+2)\frac{dy}{dx} = x^2+4x-9 \quad x \neq -2$$

and $y(0)=0$ then $y(-4)$ is equal to

(a) 0 (b) 1 (c) -1 (d) 2

$$(x+2)\frac{dy}{dx} = x^2+4x-9$$

$$\frac{dy}{dx} = \frac{x^2+4x-9}{x+2}$$

$$\frac{dy}{dx} = x+2 - \frac{13}{x+2}$$

$$\int \frac{dy}{dx} = \int\left((x+2) - \frac{13}{x+2}\right) dx$$

$$y = \frac{(x+2)^2}{2} - 13\log|x+2| + c$$

$$y(0) = 0$$

$$0 = 2 - 13\log 2 + c$$

$$c = 13\log 2 - 2$$

$$y(-4) = \cancel{2} - \cancel{13\log 2} + \cancel{13\log 2} - \cancel{2}$$

$$y(-4) = 0$$

(a)