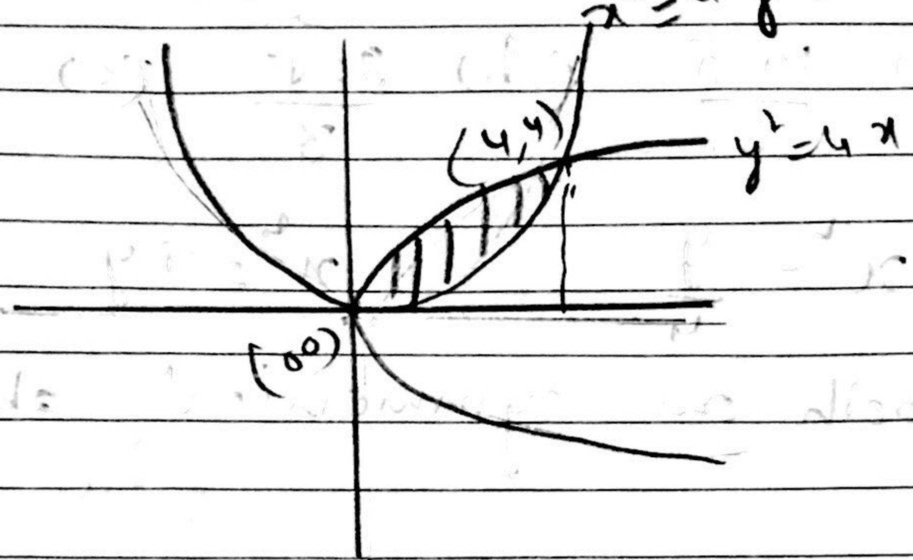
2011

The area bounded by the curve y^2
and $x^2 = 4y$ is

(a) $\frac{32}{3}$ (b) $\frac{16}{3}$ (c) $\frac{8}{3}$ (d) 0



point of intersection $= (4, 4)$

Reqd Area $= \Big($Area under the curve $y^2 =$
$\quad\ \ \Big($Area under the curve x^2

$$= \int_0^4 y\,dx - \int_0^4 y\,dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$$

2011

Let $f : [-1, 2] \to [0, \infty)$ be a cont. function such that $f(x) = f(1-x)$ $\forall\ x \in [-1, 2]$. If $R_1 = \int_{-1}^{2} x f(x)\, dx$ and R_2 are the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$ and the x-axis Then

(a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$
(c) $2R_1 = R_2$ (d) $3R_1 = R_2$

$$R_1 = \int_{-1}^{2} x f(x)\, dx \qquad \text{I}$$

$$R_1 = \int_{-1}^{2} (-1+2-x)\, f(-1+2-x)\, dx$$

$$R_1 = \int_{-1}^{2} (1-x)\, f(1-x)\, dx$$

$$R_1 = \int_{-1}^{2} (1-x)\, f(x)\, dx \qquad \text{II}$$

given $f(x) = f(1-x)$

DATE

$$R_2 = \int_{-1}^{2} f(x)\,dx \quad \text{(given)} \quad \underline{III}$$

Add $\underline{I}$ and $\underline{II}$

$$2R_1 = \int_{-1}^{2} x\,f(x)\,dx + \int_{-1}^{2} (1-x)\,f(x)\,dx$$

$$= \int_{-1}^{2} f(x)\,dx \quad \underline{IV}$$

From $\underline{III}$ and $\underline{IV}$

$$R_2 = 2R_1$$

(c)

2011

Q. The straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$ and $x = 0$ into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$.

Let R_1 area between 0 to b

R_2 area between b to 1

$$R_1 - R_2 = \frac{1}{4}$$

$$\int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \frac{1}{4}$$

$$\left[\frac{(1-x)^3}{-3}\right]_0^b - \left(\frac{(1-x)^3}{-3}\right]_b^1 = \frac{1}{4}$$

$$-\frac{1}{3}\left(\frac{(1-b)^3}{1} - 1\right) + \frac{1}{3}\left[0 - (1-b)^3\right] = \frac{1}{4}$$

$$(1-b)^3 = \frac{1}{8}$$

$$1 - b = \frac{1}{2} \qquad b = \frac{1}{2}$$

(b)

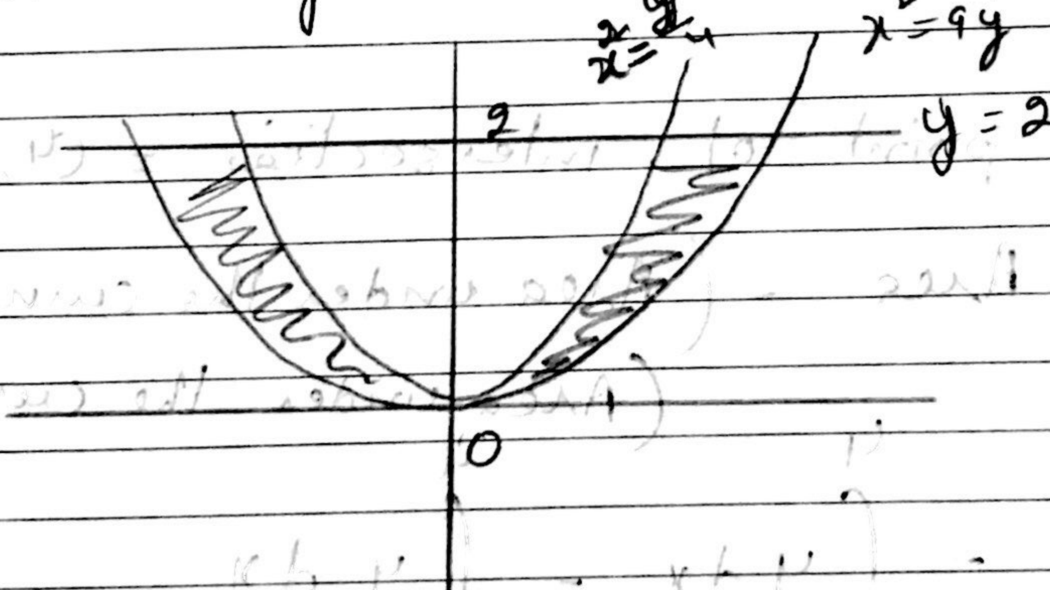
2012

The area bounded between the parabola $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is

(a) $\frac{10\sqrt{2}}{3}$ (b) $\frac{20\sqrt{3}}{3}$ (c) $10\sqrt{2}$ (d) $20\sqrt{2}$

$x^2 = \frac{y}{4}$, $x^2 = 9y$

both are symmetrical about y-axis



Required area

$2\left[\left(\text{Area under the curve } x^2 = 9y\right) - \text{Area under the curve } x^2 = \frac{y}{4}\right)$

$2\left[\int_0^2 x\,dy - \int_0^2 x\,dy\right]$

DATE

$$2\left[\int_{0}^{2}\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy\right]$$

$$2 \times \frac{10\sqrt{2}}{3} = \frac{20\sqrt{2}}{3}$$

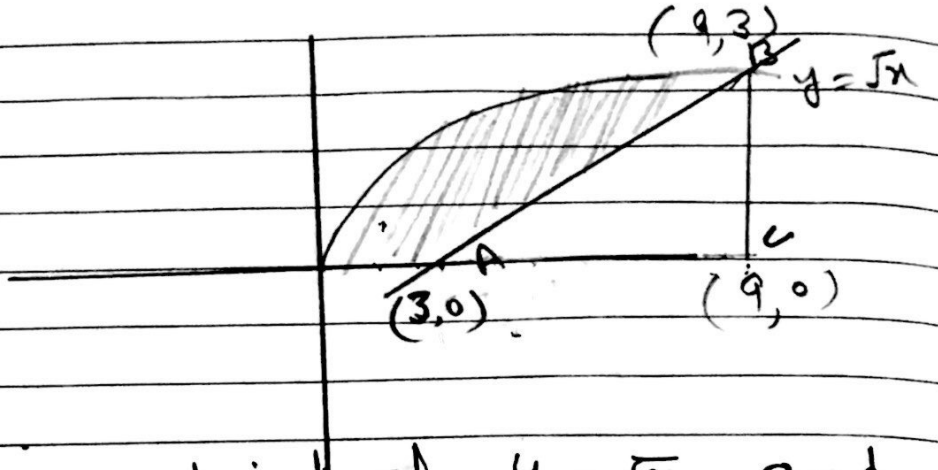
(a)

DATE

2013

The area bounded by the curve $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant

(a) $\frac{7}{32}$ (b) $\frac{5}{64}$ (c) $\frac{15}{64}$ (d) $\frac{9}{12}$



Intersecting point of $y = \sqrt{x}$ and $2y - x + 3 = 0$ is $(9,3)$

$\therefore$ $2y - x + 3 = 0$

$2\sqrt{x} - x + 3 = 0$

$2\sqrt{x} = x - 3$

$4x = x^2 + 9 - 6x$

$x^2 - 10x + 9 = 0$

$(x - 9)(x - 1) = 0$

$x = 9, \quad x = 1$

$x = 9 \Rightarrow y = 3$

$x = 1 \Rightarrow y = 1$

DATE

$2y - x + 3 = 0$

$y = 0 \Rightarrow x = 3$

line cuts x-axis at $(3,0)$

Required area

(Area under curve $y = \sqrt{x}$) - (are
line

$$\int_0^9 \sqrt{x}\,dx - \frac{1}{2} \times \cancel{6}^{3} \times 3$$

$$\left[\frac{x^{3/2}}{3/2}\right]_0^9$$

$$\frac{2}{3}(9)^{\frac{3}{2}} - 9$$

$$\frac{2}{\cancel{3}} \times \cancel{27}^{9} - 9$$

$$18 - 9$$

$$9$$

2013 online

The area bounded by the curve $y = \log x$ and the line $y = 0$, $y = \log 3$ and $x = 0$ is equal to

(a) 3 (b) $3 \log - 2$ (c) $3 \log 3 + 2$ (d) 2

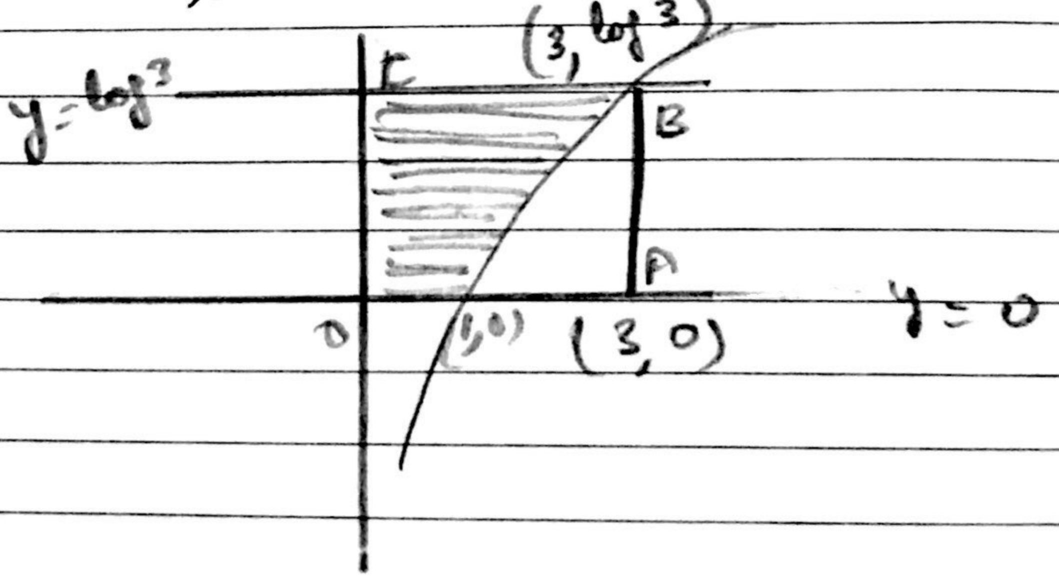
intersecting point of
$y = \log x$ and $y = \log 3$

$\log 3 = \log x \Rightarrow x = 3$

(or) $\log 3 - \log x = 0$

$\log \frac{3}{x} = 0$

$\frac{3}{x} = e^0 \qquad \frac{3}{x} = 1 \quad 3 = x$.



Required area = (Area of OABC) − (area of under the curve $y = \log x$)

$OA \times OC - \int_{1}^{3} \log x \, dx$

DATE

$3\log 3 - (x\log x - x)_1^3$

$3\cancel{\log 3} - 3\cancel{\log 3} + 3 + 1$

$-3+1 = -2$

So area $= 2$ sq unit.

d

2013 online

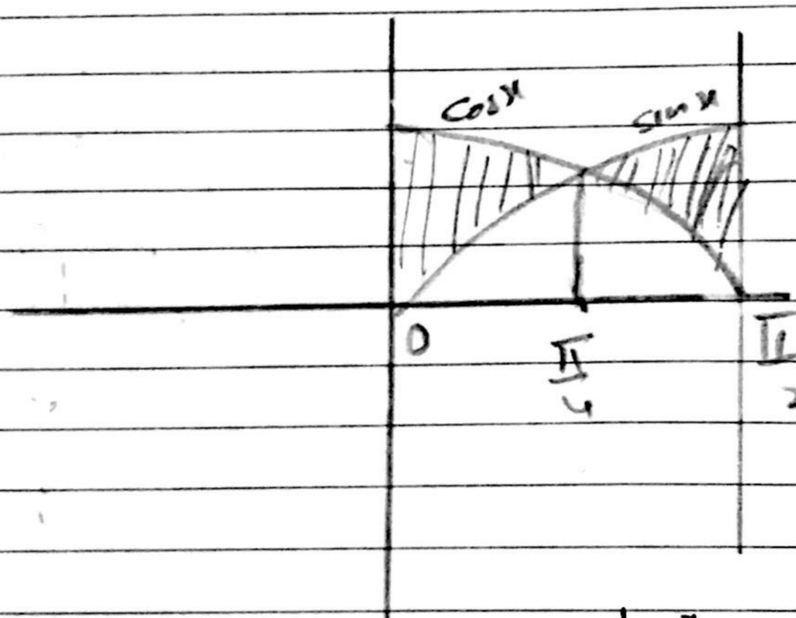
The area under the curve

$y = |\cos x - \sin x| \quad 0 \le x \le \frac{\pi}{2}$

and above x-axis

(a) $2\sqrt{2}$ (b) $2\sqrt{2} - 2$ (c) $2\sqrt{2} + 2$

(d) 0



Area of shaded portion

$$\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$\left(\sin x + \cos x\right)_0^{\pi/4} + \left(-\cos x - \sin x\right)_{\pi/4}^{\pi/2}$$

$$\frac{2}{\sqrt{2}} - 1 + \left(-1 + \frac{2}{\sqrt{2}}\right)$$

$$\sqrt{2} - 1 - 1 + \sqrt{2} = 2\sqrt{2} - 2$$ (b)

classmate

2014 offline

The area of the region describe[d] by $A = \{(x,y) : x^2+y^2 \le 1$ and $y^2 \geq 1-x\}$ is

(a) $\frac{\pi}{2}+\frac{4}{3}$ (b) $\frac{\pi}{2}-\frac{4}{3}$

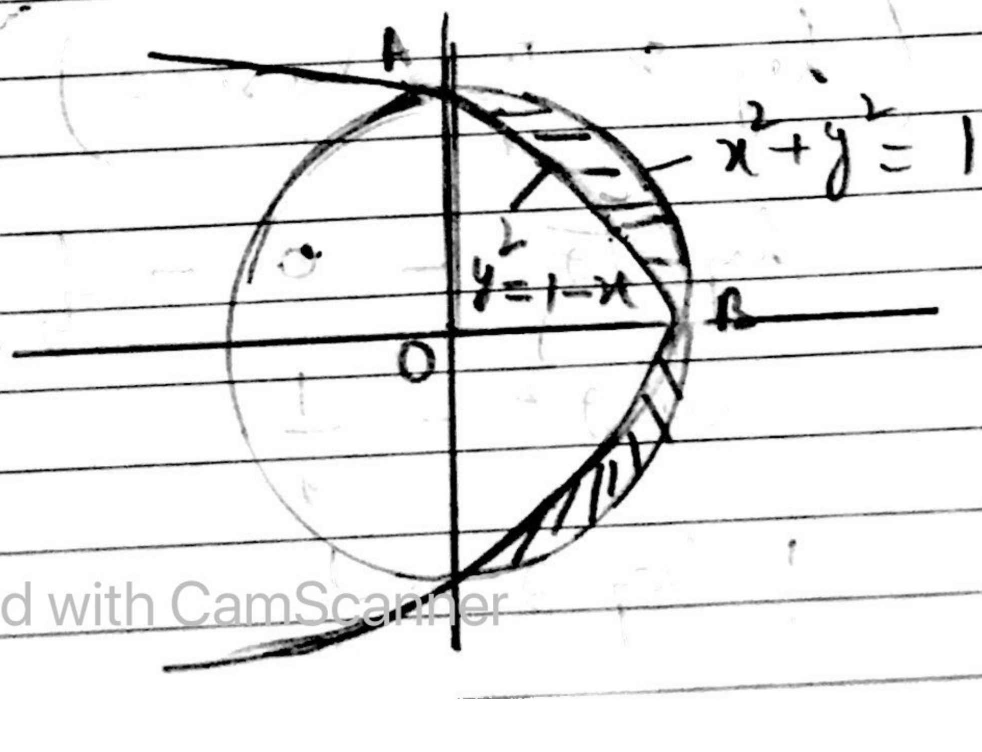
(c) $\frac{\pi}{2}-\frac{2}{3}$ (d) $\frac{\pi}{2}+\frac{2}{3}$

$x^2+y^2=1$ it is a circle with centre $(0\,0)$ an[d] radius 1

$y^2 \geq 1-x$

$y^2 = -(x-1)$

it is parabola vertex $(1,0)$



DATE

Required area

$$2\left[\left(\text{Area of the sector } AOB\right) - \left(\text{Area under the curve } y^2 = 1-x\right)\right]$$

$$2\left[\frac{\theta}{360}\pi r^2 - \int_0^1 \sqrt{1-x}\, dx\right]$$

$$2\left[\frac{90}{360}\times\pi(1)^2 - \left(\frac{2}{3}(1-x)^{3/2}\right)_0^1\right]$$

$$2\left[\frac{1}{4}\pi - \frac{2}{3}\right]$$

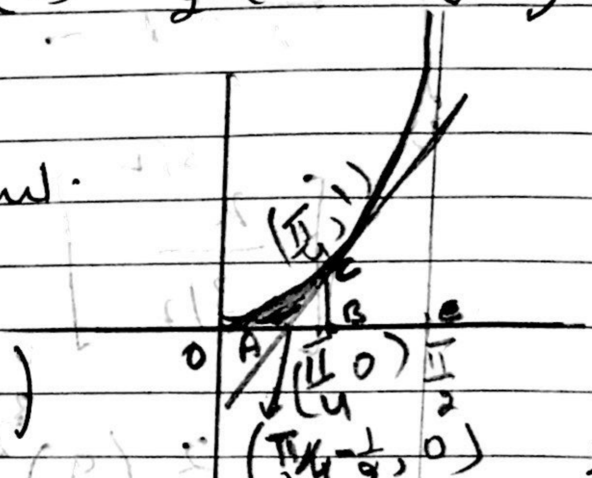
$$\frac{\pi}{2} - \frac{4}{3}$$

2014 online -

The area of the region above x-axis bounded by the curve $y = \tan x$ $0 \le x \le \frac{\pi}{4}$ and tangent to the curve at $x = \frac{\pi}{4}$ is

(a) $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$ (b) $\frac{1}{2}\left(\log 2 + \frac{1}{2}\right)$

(c) $\frac{1}{2}(1 - \log 2)$ (d) $\frac{1}{2}(1 + \log 2)$



$y = \tan x$ $\frac{dy}{dx} = \sec^2 x$

Equation of tangent at $\left(\frac{\pi}{4}, 1\right)$

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

$$\left[\because \left(\frac{dy}{dx}\right)_{x = \frac{\pi}{4}} = \sec^2 \frac{\pi}{4} = (\sqrt{2})^2 = 2\right]$$

$\therefore$ slope $= 2$

So equation of tangent

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

It will meet x-axis at

$$y = 0$$

$$-1 = 2x - \frac{\pi}{2}$$

$$\frac{\pi}{2} - 1 = 2x$$

$$\frac{\pi}{4} - \frac{1}{2} = x$$

DATE

$$AB = \left(\frac{\pi}{4} - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right)$$

$$\cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{1}{2}$$

$$\frac{1}{2}$$

Area of required region

(Area under the curve $y = \tan x$) − (Area of the ΔABC)

$$\int_0^{\pi/4} \tan x \, dx - \frac{1}{2} AB \times BC$$

$$\Big|\log \sec x\Big|_0^{\pi/4} - \frac{1}{2} \times \frac{1}{2} \times 1$$

$$\left(\log \sec \frac{\pi}{4} - \log \sec 0\right) - \frac{1}{4}$$

$$\log \sqrt{2} - 0 - \frac{1}{4}$$

$$\log 2^{\frac{1}{2}} - \frac{1}{4}$$

$$\frac{1}{2} \log 2 - \frac{1}{4}$$

$$\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$$

(a)

DATE

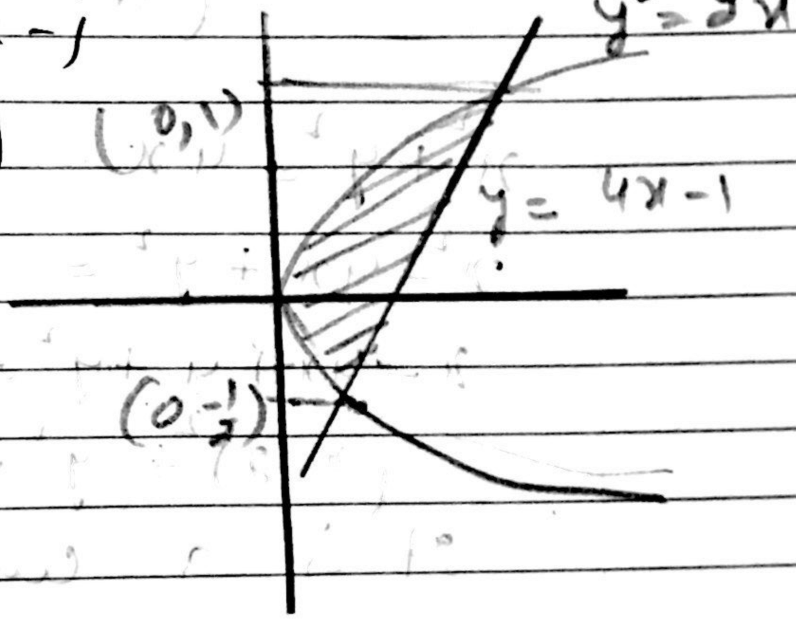
2015

The area (in square units) of the region described by $\{(x, y) : y^2 \le 2x$ and $y \ge 4x - 1\}$ is

(a) $\frac{7}{32}$ (b) $\frac{5}{64}$ (c) $\frac{15}{64}$ (d) $\frac{9}{32}$

Intersecting point of

$y^2 = 2x \quad y = 4x - 1$



$y^2 = 2\left(\frac{y+1}{4}\right)$

$y^2 = \frac{y+1}{2}$

$2y^2 = y + 1$

$2y^2 - y + 1 = 0$

$(y - 1)(y + \frac{1}{2}) = 0$

$y = 1, -\frac{1}{2}$

Required area

$$\int_{-\frac{1}{2}}^{1} \frac{1}{4}(y+1)\,dy - \int_{-\frac{1}{2}}^{1} \frac{1}{2} y^2\,dy$$

After solving $\frac{9}{32}$

(d)

2016 off line

The area (in square units) of the region $\{(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x$ $x \geq 0, y \geq 0\}$ is

(a) $\pi - \frac{4}{3}$ (b) $\pi - \frac{8}{3}$ (c) $\pi - \frac{4\sqrt{2}}{3}$

(d) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

$x^2 + y^2 = 4x$

$x^2 - 4x + y^2 = 0$

$x - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = (2)^2$

It is a circle with centre $(2,0)$ and radius 2

Intersecting point of parabola and circle i.e $x^2+y^2=4x$ & $y^2=2x$

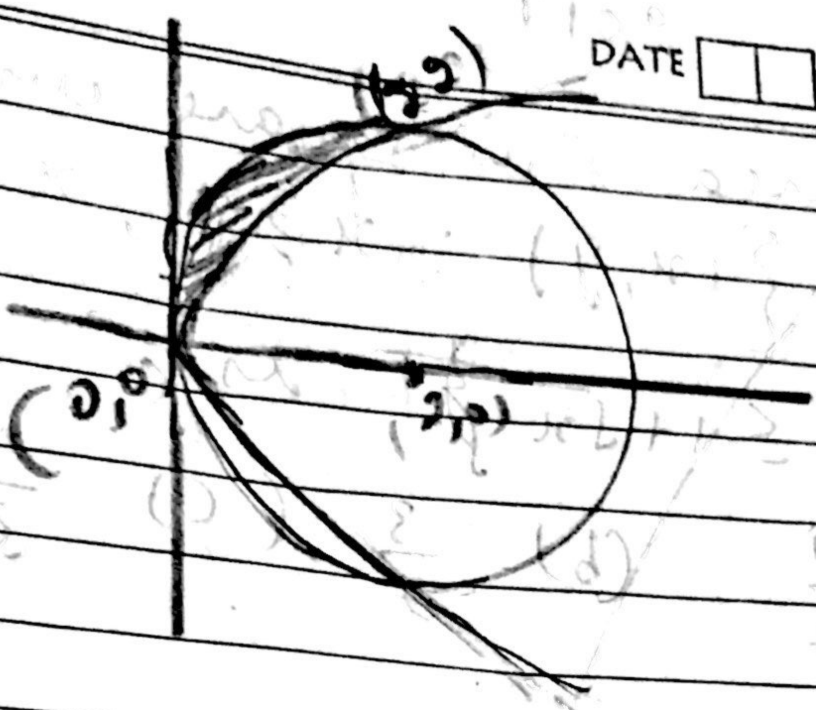
$x^2 + 2x = 4x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$x = 0$ $y = 0$, $x = 2$ $y = 2$



Required area

$$\int_0^2 \sqrt{(4x - x^2)}\, dx - \int_0^2 \sqrt{2x}\, dx$$

$$\int_0^2 \sqrt{4 - (2-x)^2}\, dx - \sqrt{2} \int_0^2 x$$

After solving

$$\pi - \frac{8}{3}$$

2016 online

The area (in square units) of the region described by

$A = \{(x,y) : y \geq x^2 - 5x + 4,\ x + y \geq 1,\ y \leq 0\}$ is

(a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$

$x + y = 1$ it is a straight line which cut x-axis and y-axis at (1, 0) and (0, 1)

$y = x^2 - 5x + 4$ it is a parabola which cut y-axis at (0, 4) and x-axis at (1, 0), (4, 0)

Intersecting point of parabola and line are

$$y = x^2 - 5x + 4$$

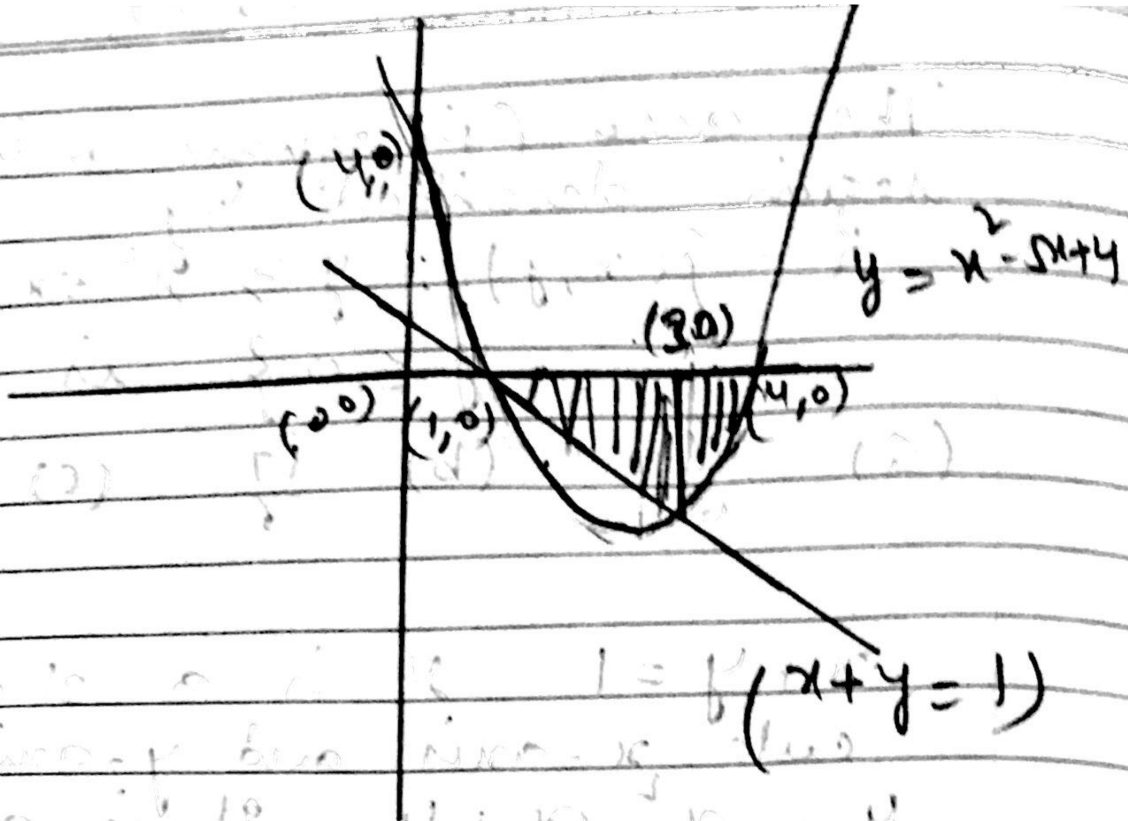
$$(1 - x) = x^2 - 5x + 4$$

$$0 = x^2 - 4x + 3$$

$$0 = (x - 3)(x - 1) \qquad x = 1, 3$$

$x = 1 \quad y = 4$

$x = 3 \quad y = -2$



Required area

$$\int_1^3 -(1-x)\,dx + \int_3^4 -(x^2 - 5x + 4)\,dx$$

$$\int_1^3 (x-1)\,dx + \int_3^4 (-x^2 + 5x - 4)\,dx$$

$$\int_1^3 (x-1)\,dx - \int_3^4 (x^2 - 5x + 4)\,dx$$

After solving $\dfrac{19}{6}$ a