2013 offline

If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are co-planar then k can have

(a) exactly three values

(b) any value (c) exactly one value

(d) exactly two values.

The two lines will be co-planar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

After solving we get.

$k = 0, -3$

(d) exactly two values

2013 off line

Distance between the parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{9}{2}$ (d) $\frac{3}{2}$

one of the point on plane $2x+y+2z=8$ is $(4,0,0)$

Now find its distance from the plane $4x+2y+4z+5=0$

$$\frac{|4\times4+0+0+5|}{\sqrt{4^2+2^2+4^2}}$$

$$\frac{21}{\sqrt{16+4+16}} = \frac{\cancel{21}^{7}}{\cancel{6}_{2}} = \frac{7}{2}.$$

(b)

2013 online

Q. The lines $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+1}{3}$ and $\frac{x+2}{2} = \frac{y-k}{3} = \frac{z}{4}$ are coplanar Then the value of k is

(a) $\frac{11}{2}$ (b) $-\frac{11}{2}$ (c) $\frac{9}{2}$ (d) $-\frac{9}{2}$

$$\frac{x-(-1)}{2} = \frac{y-1}{1} = \frac{z-(-1)}{3}$$

$$\frac{x-(-2)}{2} = \frac{y-k}{3} = \frac{z-0}{4}$$ are

co-planar if

$$\begin{vmatrix} -2+1 & k-1 & 0+1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

After solving we get.

$$k = \frac{11}{2}$$

a

2013 online

Q. Let Q be the foot of perpendicular from the origin to the plane $4x-3y+z+13=0$ and R be the point $(-1,1,-6)$ on the plane. Then length of QR is

(a) $\sqrt{14}$ (b) $\sqrt{\frac{19}{2}}$ (c) $3\sqrt{\frac{7}{2}}$ (d) $\frac{3}{\sqrt{2}}$

Sol Let the co-ordinates of Q (a_1, b_1, c_1). It lies on the plane

So $4a_1 - 3b_1 + c_1 + 13 = 0$ I

O is the origin

OQ is perpendicular to the plane

$$\frac{a_1}{4} = \frac{b_1}{-3} = \frac{c_1}{1} = k$$

$a_1 = 4k$ $b_1 = -3k$ $c_1 = k$

substitute in I

$$4(4k) - 3(-3k) + k + 13 = 0$$

$$16k + 9k + k + 13 = 0$$

$$26k + 13 = 0$$

$$k = -\frac{1}{2}$$

$$Q(a_1, b_1, c_1) = Q\left(4\times-\frac{1}{2}, -3\times-\frac{1}{2}, -\frac{1}{2}\right)$$

$$= Q\left(-2, \frac{3}{2}, -\frac{1}{2}\right)$$

$Q\left(-2, \frac{3}{2}, -\frac{1}{2}\right) \quad R(-1, 1, -6)$

$QR = \sqrt{(-2+1)^2 + \left(\frac{3}{2}-1\right)^2 + \left(-\frac{1}{2}+6\right)^2}$

$\sqrt{1 + \frac{1}{4} + \frac{121}{4}}$

$\sqrt{\frac{\cancel{126}\,63}{\cancel{4}\,2}}$

$\frac{3\sqrt{7}}{\sqrt{2}} = 3\sqrt{\frac{7}{2}}$ (C)

2013 online

The acute angle between two lines such that the direction cosines l, m, n of each of them satisfy the equations $l+m+n=0$ and $l^2+m^2-n^2=0$ is (a) 15° (b) 30° (c) 60° (d) 45°

$$l^2+m^2-n^2=0$$
$$l^2+m^2=n^2$$
$$(l+m)^2-2lm=n^2$$
$$(-n)^2-2lm=n^2$$
$$\cancel{n^2}-2lm=\cancel{n^2}$$
$$-2lm=0 \Rightarrow l=0 \text{ or } m=0$$

If $l=0$ substitute in $l+m+n=0$
$m=-n$

If $m=0$ substitute in $l+m+n=0$
$l=-n$

Direction ratios are $0, -1, 1$
$-1, 0, 1$

angle between them

$$\cos^{-1}\frac{(0\times-1)+(-1\times0)+(1\times1)}{\sqrt{2}\,\sqrt{2}}$$

$$\cos^{-1}\frac{1}{2}=60^\circ$$

c

2013 online

Q. If the lines L_1 and L_2 in space are defined by

$L_1 = \{x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1), z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\}$

and $L_2 = \{x = \sqrt{\mu}\, y + (1 - \sqrt{\mu}), z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$

Then L_1 is perpendicular to L_2 for all non negative reals λ and μ such that.

(a) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (b) $\lambda \neq \mu$

(c) $\lambda + \mu = 0$ (d) $\lambda = \mu$

$x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1), \quad z = (\sqrt{\lambda} - 1) y + \sqrt{\lambda}$

$x - (\sqrt{\lambda} - 1) = \sqrt{\lambda}\, y, \quad z - \sqrt{\lambda} = (\sqrt{\lambda} - 1) y$

$\dfrac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = y, \quad \dfrac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1} = y$

$\Rightarrow L_1 = \dfrac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = \dfrac{y}{1} = \dfrac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1} \quad \text{I}$

$x = \sqrt{\mu}\, y + (1 - \sqrt{\mu}), \quad z = (1 - \sqrt{\mu}) y + \sqrt{\mu}$

$x - (1 - \sqrt{\mu}) = \sqrt{\mu}\, y, \quad z - \sqrt{\mu} = (1 - \sqrt{\mu}) y$

$\dfrac{x - (1 - \sqrt{\mu})}{\sqrt{\mu}} = y, \quad \dfrac{z - \sqrt{\mu}}{1 - \sqrt{\mu}} = y$

$\Rightarrow L_2 = \dfrac{x - (1 - \sqrt{\mu})}{\sqrt{\mu}} = \dfrac{y}{1} = \dfrac{z - \sqrt{\mu}}{1 - \sqrt{\mu}} \quad \text{II}$

L_1 is perpendicular to L_2

$\sqrt{\lambda}\sqrt{\mu} + (1)(1) + (\sqrt{\lambda} - 1)(1 - \sqrt{\mu}) = 0$

$\cancel{\sqrt{\lambda\mu}} + \cancel{1} + \sqrt{\lambda} - \cancel{\sqrt{\lambda\mu}} - \cancel{1} + \sqrt{\mu} = 0$

$\sqrt{\lambda} + \sqrt{\mu} = 0$

$\Rightarrow \lambda = \mu$

(d)

2013 online

Q 27. The projections of a line segment on the x, y and z-axis in 3 dimensional space are 2, 3 and 6 respectively. Then the length of the line segment

(a) 12 (b) 7 (c) 9 (d) 6

Sol Required length of the line segment.

$$\sqrt{2^2+3^2+6^2}$$

$$\sqrt{4+9+36}$$

$$\sqrt{49} = 7$$

b

2013 online

Q Let ABC be a triangle with ve
at points $A(2,3,5)$ $B(-1,3,2$
$C(\lambda, 5, \mu)$ in three dimension
If the median through A
equally inclined with the
then (λ, μ) is equal to

(a) $(10, 7)$. (b) $(7, 5)$ (c) $(7, 10$

Sol Let AD is the median
co-ordinates of D $\left(\frac{\lambda-1}{2}, \frac{5+}{2}\right.$
(mid poi
D.R of AD $\left(\frac{\lambda-1}{2} - 2, 4-3, \right.$

$\left(\frac{\lambda-5}{2}, 1, \frac{\mu-}{2}\right.$

AD is equally inclined
D.R of AD $(1, 1, 1$

$\Rightarrow \frac{\lambda-5}{2} = 1 \qquad \frac{\mu-8}{2} =$

$\lambda = 7 \qquad \mu = 10$

$(\lambda, \mu) = (7, 10)$

2013 online

Q The equation of a plane through the line of intersection of the planes $x+2y=3$, $y-2z+1=0$ and perpendicular to the first plane is

(a) $2x-y-10z=9$ (b) $2x-y+7z=11$
(c) $2x-y+10z=11$ (d) $2x-y-9z=10$

Sol Equation of plane through the intersection of the given planes is

$$(x+2y-3)+\lambda(y-2z+1)=0 \quad \text{I}$$

$$x+(2+\lambda)y-2\lambda z+(\lambda-3)=0$$

It is perpendicular to first plane i.e on $x+2y-3=0$

$$1+2(2+\lambda)+(-2\lambda)(0)=0$$

$$1+4+2\lambda=0$$

$$2\lambda=-5$$

$$\lambda=-\frac{5}{2}$$

substitute this value in I

$$(x+2y-3)-\frac{5}{2}(y-2z+1)=0$$

$$2x+4y-6-5y+10z-5=0$$

$$2x-y+10z=11 \quad \text{(c)}$$

2014 online

Q A line in 3-dimensional space makes an angle θ $(0 < \theta \le \frac{\pi}{2})$ with both the x and y axes. Then the set of all values of θ in the interval

(a) $\left(0, \frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$

Sol A line makes an angle θ with x and y-axis

Suppose it makes an angle α with z-axis

Direction cosines are

$\cos\theta, \cos\theta, \cos\alpha$.

$\cos^2\theta + \cos^2\theta + \cos^2\alpha = 1$

$2\cos^2\theta = 1 - \cos^2\alpha$

$2\cos^2\theta = \sin^2\alpha$

$0 \le \sin^2\alpha \le 1$

$0 \le 2\cos^2\theta \le 1$

$0 \le \cos^2\theta \le \frac{1}{2}$

$0 \le \cos\theta \le \frac{1}{\sqrt{2}}$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (c)

2014 online

Q Let $A(2,3,5)$, $B(-1,3,2)$, $C(\lambda,5,\mu)$ be the vertices of triangle ABC. If the median through A is equally inclined to the co-ordinate axes then

(a) $5\lambda - 8\mu = 0$ (b) $8\lambda - 5\mu = 0$

(c) $10\lambda - 7\mu = 0$ (d) $7\lambda - 10\mu = 0$

$B(-1,3,2)$ $C(\lambda,5,\mu)$

Sol MID point of BC $D\left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

DR of AD $\left(\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-\right.$

$\left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right)$

Median is equally inclined to a

$\frac{\lambda-5}{2} = 1$ $\frac{\mu-8}{2} = 1$

$\lambda = 7$ $\mu = 10$

It satisfies option c which

is $10\lambda - 7\mu = 0$

(c)

2014 online

The plane containing the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$ passes through the point.

(a) $(1,-2,5)$ (b) $(1,0,5)$ (c) $(0,3,-5)$

(d) $(-1,-3,0)$

Sol. Equation of plane containing the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is

$$a(x-1) + b(y-2) + c(z-3) = 0$$

where $a + 2b + 3c = 0$

line $\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$ parallel to

plane $a(x-1)+b(y-2)+c(z-3)=0$

$$a + b + 4c = 0 \qquad \text{I}$$

$$a + 2b + 3c = 0 \qquad \text{II}$$

$$\begin{array}{cccc} 1 & 4 & 1 & 1 \\ 2 & 3 & 1 & 2 \end{array}$$

$$\frac{a}{3-8} = \frac{b}{4-3} = \frac{c}{2-1}$$

$$\frac{a}{-5} = \frac{b}{1} = \frac{c}{1}$$

Equation of plane

$$-5(x-1) + 1(y-9) + (z-3) = 0$$

$$-5x + 5 + y - 9 + z - 3 = 0$$

$$-5x + y + z = 0$$

This plane passes through the point

$(1, 0, 5)$

(b)

2014 online

Q A symmetrical form of the line of intersection of the planes

$x = ay + b$ and $z = cy + d$ is

(a) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$

(b) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

(c) $\frac{x-a}{b} = \frac{y-0}{1} = \frac{z-c}{d}$

(d) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$.

Sol $x = ay + b \qquad z = cy + d$

$x - b = ay \qquad z - d = cy$

$\frac{x-b}{a} = y \qquad \frac{z-d}{yc} = y$

$\frac{x-b}{a} = y = \frac{z-d}{c}$

$\frac{x-b}{a} - 1 = y - 1 = \frac{z-d}{c} - 1$

$\frac{x-b-a}{a} = y - 1 = \frac{z-d-c}{c}$

(d)

Q- The distance between the planes $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7 then d is

(a) 41 or -42 (b) 42 or -43 (c) -41 or 43

(d) -42 or 44

sol
$$\frac{|d-1|}{\sqrt{4^2+(-2)^2+(-4)^2}} = 7$$

$$\frac{|d-1|}{\sqrt{16+4+16}} = 7$$

$$\frac{|d-1|}{6} = 7$$

~~d-1=43~~

$$d-1 = \pm 42$$

$$d = 43, -41$$

(c)

2014 online

If the angle between the line $2(x+1) = y = z+4$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is $\frac{\pi}{6}$

Then the value of λ is

(a) $\frac{135}{7}$ (b) $\frac{45}{11}$ (c) $\frac{45}{7}$ (d) $\frac{135}{11}$

$$2(x+1) = y = z+4)$$

$$\cancel{2}\frac{(x+1)}{\cancel{2}} = \frac{y}{2} = \frac{z+4}{2}$$

$$\frac{x+1}{1} = \frac{y}{2} = \frac{z+4}{2} \quad \text{I}$$

Angle between line I and plane

$2x - y + \sqrt{\lambda}\, z + 4 = 0$ is $\frac{\pi}{6}$

$$\Rightarrow \sin\frac{\pi}{6} = \frac{\left|(2)(1) + (-1)(2) + \sqrt{\lambda} \times 2\right|}{\left(\sqrt{1^2+2^2+2^2}\right)\left(\sqrt{2^2+(-1)^2+(\sqrt{\lambda})^2}\right)}$$

$$\frac{1}{2} = \frac{\left|\cancel{2} - \cancel{2} + 2\sqrt{\lambda}\right|}{\sqrt{9}\sqrt{4+1+\lambda}}$$

$$\sqrt{9}\left(\sqrt{5+\lambda}\right) = 2\left(2\sqrt{\lambda}\right)$$

$$9(5+\lambda) = \cancel{4} \times 4\lambda$$

$$45 + 9\lambda = 16\lambda$$

$$45 = 7\lambda$$

$$\frac{45}{7} = \lambda \qquad (c)$$

2015 offline

Q. The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is

(a) $2\sqrt{14}$ (b) 8 (c) $2\sqrt{21}$ (d) 13

Sol Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3k+2, 4k-1, 12k+2)$

This point lie on the plane $x-y+z=16$

$3k+2 - (4k-1) + (12k+2) = 16$

$3k+2 - 4k+1 + 12k+2 = 16$