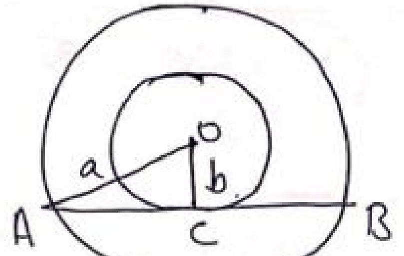
SECTION A

Question No 1 to 6 carry one mark each

1. Two concentric circles of radius a and b (a > b) are given. Find the length of the chord of larger circle which touches the smaller circle



$OC \perp AB$

$$AC^2 = OA^2 - OC^2$$
$$= a^2 - b^2$$
$$AC = \sqrt{a^2 - b^2}$$

Perpendicular from the centre to the chord bisect the chord

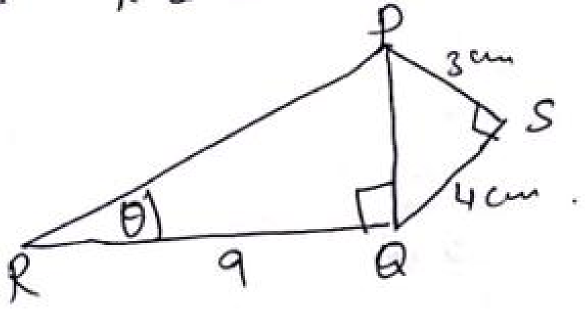
AB = length of the chord =

$$2\,AC$$
$$2\sqrt{a^2-b^2}$$

2. In fig $PS = 3\,cm$ $QS = 4\,cm$

$\angle PRQ = \theta$ $\angle PSQ = 90^\circ$ $PQ \perp RQ$

and $RQ = 9\,cm$ Evaluate $\tan\theta$



$\angle PSQ = 90^\circ$

$PQ^2 = PS^2 + SQ^2$

$= 3^2 + 4^2$

$= 9 + 16$

$= 25$

$PQ = 5$

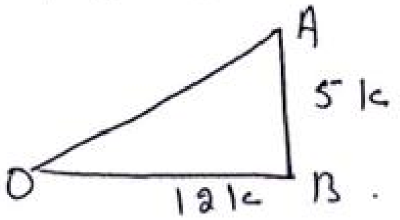
$RQ = 9$

$$\tan\theta = \frac{PQ}{RQ} = \frac{5}{9}$$

or.

Q. $\tan\alpha = \frac{5}{12}$

Find the value of $\sec\alpha$.



$\tan\alpha = \frac{5}{12}$

$AB = 5k$ $OB = 12k$

$OA^2 = AB^2 + OB^2$

$= (5k)^2 + (12k)^2$

$$OA^2 = 25k^2 + 144k^2$$

$$OA^2 = 169k^2$$

$$OA = 13k$$

$$\sec\alpha = \frac{OA}{OB}$$

$$= \frac{13\cancel{k}}{12\cancel{k}}$$

$$= \frac{13}{12}.$$

3. Write the discriminant of the quadratic equation

$$(x+5)^2 = 2(5x-3)$$

$$x^2 + 25 + \cancel{10x} = \cancel{10x} - 6$$

$$x^2 + 25 + 6 = 0$$

$$x^2 + 31 = 0$$

Discriminant $= b^2 - 4ac$

$$(0)^2 - 4 \times 1 \times 31$$

$$0 - 124$$

$$-124.$$

4. Find after how many places of decimal the decimal form of the number $\frac{27}{2^3 \times 5^4 \times 3^2}$ will terminate

Sol $\frac{27}{2^3 \times 5^{4} \times 3^2}$

$\frac{\cancel{27}\ 3}{2^3 \times 5^4 \times \cancel{9}}$

$\frac{3 \times 2}{2^3 \times 5^4 \times 2} = \frac{6}{2^4\ 5^4} = \frac{6}{10^4} = .0006$

or.

Express 429 as product of prime factors.

$429 = 3 \times 11 \times 13$

$$\begin{array}{r|l} 3 & 429 \\ \hline 11 & 143 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

3, 11, 13 are prime numbers

So 429 can be pressed as product of primes.

3. Find the sum of first 10 multiples of 6.

6, 12, 18 . . . . (10) terms

$a = 6 \qquad d = 12 - 6 = 6$

$n = 10$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}\left(2 \times 6 + (10-1)6\right)$

$= 5(12 + 9 \times 6)$

$= 5(12 + 54)$

$= 5 \times 66$

$= 330$

6. Find the positive value of m for which the distance between the points A (5, -3) and B (13, m) is 10 units

According to question
distance between A and B = 10 units

$$\sqrt{(13-5)^2 + (m+3)^2} = 10$$

$$\sqrt{(8)^2 + (m+3)^2} = 10$$

Squaring both side

$$(8)^2 + (m+3)^2 = 100$$

$$64 + m^2 + 9 + 6m = 100$$

$$m^2 + 6m + 73 = 100$$

$$m^2 + 6m - 27 = 0$$

$$m^2 + 9m - 3m - 27 = 0$$

$$m(m+9) - 3(m+9) = 0$$

$$(m+9)(m-3) = 0$$

$$m = -9 \quad m = 3.$$

Positive value of m = 3.

SECTION B.

Q. No 7 to carry two marks each.

7. A die is thrown once. Find the probability of getting

I a composite number II a prime number.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(\text{a composite number}) = \frac{2}{6} = \frac{1}{3}$$

[∵ composite numbers are 4 & 6]

$$P(\text{a prime number}) = \frac{3}{6} = \frac{1}{2}$$

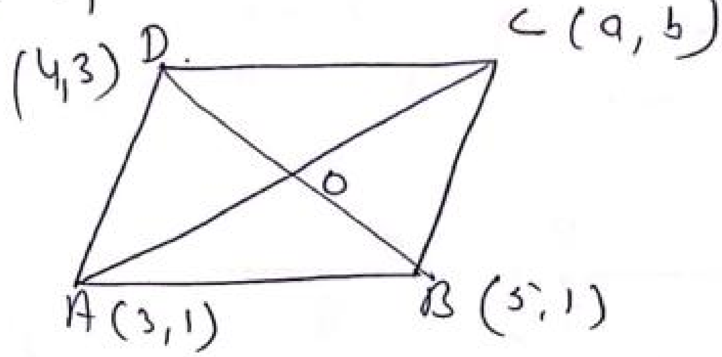
[∵ prime numbers are 2, 3, 5]

(8) Cards number 7 to 40 were put in a box. Poonam selects a card at random what is the probability that Poonam selects a card which is a multiple of seven.

Total no of cards = 34.

P (of getting a card which is multiple of seven) $= \frac{5}{34}$

9. Point A (3, 1) B (5, 1) C (a, b) D (4, 3) are the vertices of a parallelogram ABCD Find the value of a and b.



ABCD is a parallelogram.

Diagonal AC and BD bisect each other

mid point of AC = mid point of BD

$$\left(\frac{a+3}{2}, \frac{b+1}{2}\right) = \left(\frac{4+5}{2}, \frac{3+1}{2}\right)$$

$$\frac{a+3}{2} = \frac{9}{2} \qquad \frac{b+1}{2} = \frac{4}{2}$$

$$2a + 6 = 18 \qquad \frac{b+1}{2} = 2$$

$$2a = 18 - 6 \qquad b + 1 = 4$$

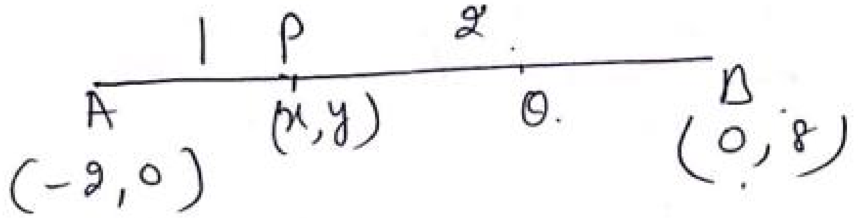
$$2a = 12 \qquad b = 3.$$

$$a = 6$$

$a = 6 \quad b = 3.$

or

Points P and Q trisect the line segment joining the points A (−2, 0) and B (0, 8) such that P is near to A. Find the co-ordinates of the point P and Q.

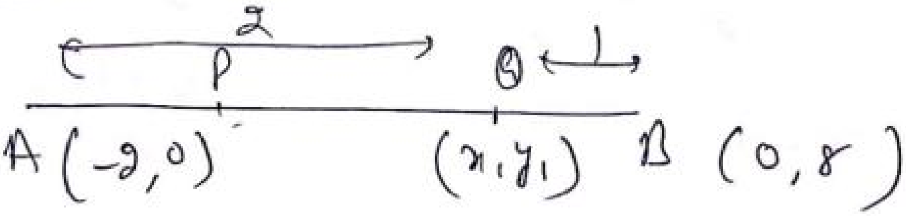


In First case Ratio is = AP : PB = 1 : 2.

$$x = \frac{1 \times 0 + 2 \times -2}{1 + 2} = -\frac{4}{3}$$

$$y = \frac{1 \times 8 + 2 \times 0}{1 + 2} = \frac{8}{3}$$

In Second case.



$$AQ : QB = 2 : 1$$

$$x_1 = \frac{2 \times 0 + 1 \times -2}{2 + 1} = \frac{-2}{3}$$

$$x_2 = \frac{2 \times 8 + 1 \times 0}{2 + 1} = \frac{16}{3}$$

10. Solve the following pair of linear equation

$3x - 5y = 4$

$9y + 7 = 9x$.

Substitution method.

$3x - 5y = 4$

$3x = 4 + 5y$

$x = \frac{4 + 5y}{3}$

Put this value in $9y + 7 = 9x$.

$9y + 7 = \cancel{9}^{3}\left(\frac{4 + 5y}{\cancel{3}}\right)$

$9y + 7 = 12 + 15y$

$9y - 15y = 12 - 7$

$-13y = 5$

$y = -\frac{5}{13}$

$x = \frac{4 + 5y}{3} = \frac{4 + 5 \times -\frac{5}{13}}{3}$

$= \frac{4 - \frac{25}{13}}{3} = \frac{\frac{52 - 25}{13}}{3}$

$= \frac{\cancel{27}^{9}}{13} \times \frac{1}{\cancel{3}} = \frac{9}{13}$

11. Q. HCF of 65 and 117 is expressible in the form $65n - 117$. Then find the value of n.

$$65 = 13 \times 5$$

$$117 = 13 \times 3 \times 3$$

$$HCF = 13.$$

$$13 = 65n - 117$$

$$13 + 117 = 65n$$

$$130 = 65n$$

$$2 \; \cancel{10} \; \frac{\cancel{130}}{\cancel{65}} = n$$

$$\cancel{5}$$

$$2 = n.$$

19 In the quadratic equation $kx^2 - 6x - 1 = 0$ determine the value of k for which the equation does not have any real root

$$kx^2 - 6x - 1 = 0$$

equation does not have real root

$$b^2 - 4ac < 0$$

$$(-6)^2 - 4(k)(-1) < 0$$

$$36 + 4k < 0$$

$$4k < -36$$

$$k < -\frac{36}{4}$$

$$k < -9$$

SECTION C

13) Question number 13 to 22 carry 3 marks each:

A, B, C are interior angles of a triangle ABC show that -

I $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

II If $\angle A = 90^\circ$ Then find the value of $\tan\left(\frac{B+C}{2}\right)$

I show $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

L.H.S $\sin\left(\frac{B+C}{2}\right)$

$A+B+C = 180 \qquad B+C = 180-A$

$\sin\left(\frac{180-A}{2}\right)$

$\sin\left(\frac{180}{2}-\frac{A}{2}\right)$

$\sin\left(90-\frac{A}{2}\right)$

$\cos\frac{A}{2} = RHS$

II $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{180-A}{2}\right)$

$= \tan\left(\frac{180}{2}-\frac{A}{2}\right)$

$= \tan\left(90-\frac{A}{2}\right)$

$= \cot\frac{A}{2}$

$= \cot\frac{90}{2} = \cot 45^\circ = 1$

B If $\tan(A+B) = 1$ or and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$0 < A+B < 90$

$A > B$

Then find the values of A and B.

$\tan(A+B) = 1$

$\tan(A+B) = \tan 45$

$A+B = 45$

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A-B) = \tan 30^\circ$

$A-B = 30$

$A + \cancel{B} = 45$

$A - \cancel{B} = 30$

$2A = 75$

$A = \frac{75}{2} = 37\frac{1}{2}^\circ$

$A - B = 30$

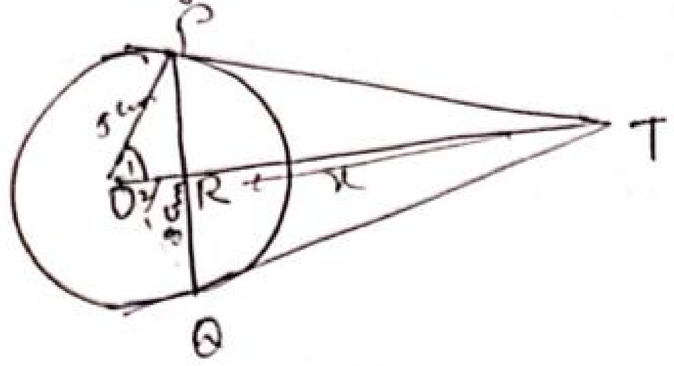
$\frac{75}{2} - B = 30$

$\frac{75}{2} - 30 = B$

$\frac{15}{2} = B$

$B = \frac{15}{2} = 7\frac{1}{2}^\circ$

14. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP



In Δ OPT and Δ OQT

OP = OQ (radius of a circle)

PT = QT

OT = OT (common)

$\Delta OPT \cong \Delta OQT$

$\angle 1 = \angle 2$ I

In Δ OPR + OQR

OP = OQ. (radius of a circle.

OR = OR (common)

$\angle 1 = \angle 2$ (by I)

$\Delta OPR \cong \Delta OQR$

$\angle ORP = \angle ORQ$

$\angle ORP + \angle ORQ = 180$

$2\angle ORP = 180$

$\angle ORP = 90$

$OR \perp PQ$

Perpendicular from the centre to the chord bisect the chord.

$\Rightarrow PR = 4cm.$

In right angled triangle ORP.

$$OP^2 = OR^2 + PR^2$$
$$5^2 = OR^2 + 4^2$$
$$OR^2 = 25 - 16$$
$$OR^2 = 9 \qquad OR = 3.$$

Let $RT = x \qquad OT = x + 3$

In right angled triangle ORT, $\angle P = 90^\circ$

$$OT^2 = OP^2 + PT^2$$
$$(x+3)^2 = 5^2 + PT^2$$
$$x^2 + 9 + 6x - 25 = PT^2$$
$$x^2 + 6x - 16 = PT^2 \quad \text{I}$$

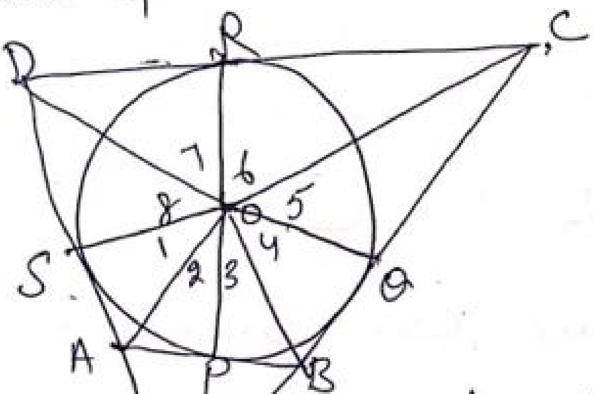
In right angled triangle PRT, $\angle R = 90^\circ$

$$PT^2 = PR^2 + RT^2$$
$$PT^2 = 4^2 + x^2$$
$$PT^2 = 16 + x^2 \quad \text{II}$$

From I and II

$$\cancel{x^2} + 6x - 16 = 16 + \cancel{x^2}$$
$$6x - 32 = 0$$
$$2(3x - 16) = 0$$
$$3x - 16 = 0$$
$$x = \frac{16}{3}.$$

14 Prove that ~~the~~ opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Given :- A quadrilateral ABCD circumscribing a circle is given.

To prove :- $\angle AOB + \angle COD = 180$

$\angle AOD + \angle BOC = 180°$

Construction :- Join OP, OQ, OR & OS.

Proof :- we know two tangents drawn from an external point to a circle subtend equal angle at the centre.

$\Rightarrow \angle 1 = \angle 2, \quad \angle 3 = \angle 4, \quad \angle 5 = \angle 6, \quad \angle 7 = \angle 8$ I

we know

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360$ (angle at the centre)

$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360$

$2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360$

$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360$$

$$\angle 2 + \angle 3 + \angle 6 + \angle 7 = \frac{\cancel{360}}{\cancel{2}}\ 180$$

$$(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180$$

$$\angle AOB + \angle COD = 180.$$

Similarly we can prove.

$$\angle BOC + \angle AOD = 180^\circ$$