Question number 1 to 10 are multiple choi[ce] questions of 1 mark each You have to select the correct choice

I The two lines $x = ay + b$, $z = cy +$ [d] and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if

(a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$

[(c)] $aa' + cc' = 1$ (d) $aa' + cc' = -1$

$$x = ay + b, \quad z = cy + d$$

$$\Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

$$x = a'y + b' \quad z = c'y + d'$$

$$\Rightarrow \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$$

They are perpendicular to each other

$$\Rightarrow aa' + 1 \times 1 + cc' = 0$$

$$aa' + cc' = -1$$

(d)

2. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$

Then value of x is

(a) 3 (b) 0 (c) -1 (d) 1

$$2(x-9x) - 3(x-4x) + 2(9x-4x) + 3 = 0$$

$$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$$

$$-16x + 9x + 10x + 3 = 0$$

$$3x + 3 = 0$$

$$3x = -3$$

$$x = -1$$

(c)

③ In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which Z_{max} occurs is

(a) 0 (b) 2 (c) finite (d) infinite.

If the objective function $z = ax + by$ has the same maximum value on two corner points of feasible region then the number of points at which Z_{max} occurs is infinite

(d)

(4) From the set $\{1, 2, 3, 4, 5\}$ numbers a and b $(a \neq b)$ are at random. The probability $\frac{a}{b}$ is an integer is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d)

$S = \{(1,2)\ (1,3)\ (1\,4)\ (1\,5).(2\,1)\ (2\,3)\ (2$
$(3,1)\ (3\,2)\ (3\,4)\ (3\,5)\ (4,1)\ (4$
$(4,5)\ (5,1)\ (5\,2)\ (5\,3)\ (5\,4)\}$

$n(S) = 20$

$A = \frac{a}{b}$ is an integer

$A = \{(2,1)\ (3,1)\ (4,1)\ (5,1)$
$(4,2)\}$.

$n(A) = 5$

Required probability $= \frac{5}{20}$

(b)

5. $\int_0^{\pi/8} \tan^2 8x\,dx$ is equal to

(a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$

$$\int_0^{\pi/8} \tan^2 8x\,dx$$

Let $8x = t$ $\quad x = 0 \Rightarrow t = 0$

$8\,dx = dt$ $\quad x = \frac{\pi}{8} \Rightarrow t = \frac{\pi}{4}$

$$\frac{1}{8}\int_0^{\pi/4} \tan^2 t\,dt$$

$$\frac{1}{8}\int_0^{\pi/4} (\sec^2 t - 1)\,dt$$

$$\frac{1}{8}\left[\int_0^{\pi/4} \sec^2 t\,dt - \int_0^{\pi/4} 1\,dt\right]$$

$$\frac{1}{8}\left[(\tan t)_0^{\pi/4} - [t]_0^{\pi/4}\right]$$

$$\frac{1}{8}\left[\tan\frac{\pi}{4} - \tan 0 - \frac{\pi}{4} + 0\right]$$

$$\frac{1}{8}\left(1 - \frac{\pi}{4}\right)$$

$$\frac{4-\pi}{8}$$

(a)

(6) If $\vec{a} \cdot \vec{b} = \frac{1}{2} |\vec{a}| |\vec{b}|$ then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 0° (b) 30° (c) 60° (d) 90°

$$\vec{a} \cdot \vec{b} = \frac{1}{2} |\vec{a}| |\vec{b}|$$

$$|\vec{a}| |\vec{b}| \cos\theta = \frac{1}{2} |\vec{a}| |\vec{b}|$$

$$\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} = 60^\circ$$

7 A bag contains 3 white, 4 black, [illegible]
balls & 2 balls are drawn at [illegible]
(without replacement) then the prob[illegible]
that both the balls are white i[illegible]

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{2[illegible]}$

There are 3 white 4 black and 2 red ba[illegible]

P(getting white ball) = $\frac{3}{9}$ = [illegible]

ball is not replaced.
so there are two white balls in the bag

Total no of balls = 8
P(getting white ball) = [illegible]

P(getting two white balls)
$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

(c)

(8) The value of

$$\tan^{-1}\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] \text{ is}$$

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$ (c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

$$\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3} = \theta.$$

$$\cos^{-1}\frac{\sqrt{5}}{3} = 2\theta \qquad \frac{\sqrt{5}}{3} = \cos 2\theta.$$

$$\cos 2\theta = \frac{\sqrt{5}}{3}$$

$$1 - 2\sin^2\theta = \frac{\sqrt{5}}{3}$$

$$2\sin^2\theta = 1 - \frac{\sqrt{5}}{3} = \frac{3-\sqrt{5}}{3}$$

$$\sin\theta = \sqrt{\frac{3-\sqrt{5}}{6}}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - \left(\sqrt{\frac{3-\sqrt{5}}{6}}\right)^2$$

$$= 1 - \frac{3-\sqrt{5}}{6}$$

$$= \frac{6-(3-\sqrt{5})}{6}$$

$$= \frac{6-3+\sqrt{5}}{6}$$

$$= \frac{3+\sqrt{5}}{6}$$

$$\cos\theta = \sqrt{\frac{3+\sqrt{5}}{6}}$$

$$\tan\theta = \sqrt{\frac{\frac{3-\sqrt{5}}{6}}{\frac{3+\sqrt{5}}{6}}}$$

$$\tan\theta = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}}$$

$$\tan\theta = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}}$$

$$\tan\theta = \frac{3-\sqrt{5}}{2}.$$

$$\theta = \tan^{-1}\frac{3-\sqrt{5}}{2}$$

$$\tan^{-1}\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$$

$$\tan^{-1}(\theta)$$

$$\tan\left(\tan^{-1}\frac{3-\sqrt{5}}{2}\right)$$

$$\frac{3-\sqrt{5}}{2}.$$

4. $A = \begin{bmatrix} a & 0 & b \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ Then det. (adj A) equal

(a) a^{27} (b) a^{9} (c) a^{6} (d) a^{2}

$A_{11} = a^2$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = 0$ $\quad A_{22} = a^2$ $\quad A_{23} = 0$

$A_{31} = 0$ $\quad A_{32} = 0$ $\quad A_{33} = a^2$

$$\text{A adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}'$$

$$= \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}$$

$$\text{adj } A = \begin{vmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{vmatrix}$$

$$\text{det. (Adj } A) = a^2 (a^4 - 0)$$

$$= a^6$$

(c)

10. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is par

to the plane

(a) $2x+3y+4z=0$ (b) $3x+4y-5z=7$

(c) $2x+y-2z=0$ (d) $x-y+z=2$

Solution = (c) $2x+y-2z=0$

because –

direction ratio of line = (3, 4,

Direction ratio of the norma

to the plane is, (2, 1,

$$3 \times 2 + 4 \times 1 - 5 \times 2 = 0$$

$$6 + 4 - 10 = 0$$

which is true

Hence line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

parallel to the plane $2x+y-2$

In Q Nos 11 to 15 fill in the blanks with correct word/ sentence :

11. The slope of the tangent to the curve $y = x^3 - x$ at the point $(2, 6)$ is ___

$$y = x^3 - x$$

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\left(\frac{dy}{dx}\right)_{(2,6)} = 3(2)^2 - 1$$

$$= 3 \times 4 - 1$$

$$= 12 - 1$$

$$= 11$$

11 The rate of change of the area of a circle with respect to ~~the~~ its radius r, when r = 3cm is

Area of circle = πr^2

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\left(\frac{dA}{dr}\right)_{r=3} = 2\pi(3)$$

$$= 6\pi$$

12. Q. $f : R \to R$ be given by $f(x) = (3 - x^3)^{1/3}$ Then fof

$$f(x) = \sqrt[3]{3-x^3}$$

$$fof(x) = f(f(x))$$

$$= f(\sqrt[3]{3-x^3})$$

$$= \sqrt[3]{3-(\sqrt[3]{3-x^3})^3}$$

$$= \sqrt[3]{3-(3-x^3)}$$

$$= \sqrt[3]{3-3+x^3}$$

$$= \sqrt[3]{x^3}$$

$$= x.$$

13. If $\vec{a}$ is a non zero vector, then $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$ equals ______

$$(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$$

$$\vec{a}\cdot\hat{i}\cdot\hat{i} + \vec{a}\cdot\hat{j}\cdot\hat{j} + \vec{a}\cdot\hat{k}\cdot\hat{k}$$

$$\vec{a} + \vec{a} + \vec{a}$$

$$3\vec{a}$$

13 The projection of the ve[ctor $\hat{i} - \hat{j}$]
on the vector $\hat{i} + \hat{j}$

Let $\vec{a} = \hat{i} - \hat{j}$

$\vec{b} = \hat{i} + \hat{j}$

Projection of vector $\vec{a}$ [on]
vector $\vec{b}$ is giv[en by]

$$\frac{1}{|\vec{b}|} (\vec{a} \cdot \vec{b})$$

$$= \frac{1 \times 1 + (-1)(1)}{\sqrt{1^2 + 1^2}}$$

$$= \frac{1 - 1}{\sqrt{2}} = 0$$

14 Q. $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$

Then $x \cdot y =$ ______

$$x + y = 2$$
$$x - y = 4$$
$$2x = 6$$
$$x = 3$$

$$x + y = 2$$
$$3 + y = 2$$
$$y = 2 - 3 = -1$$

$$x \cdot y = 3 \times (-1) = -3$$

15. If $f(x) = x|x|$ then $f'(x) =$ ______

$$f(x) = x^2 \quad \text{if } x \geq 0$$
$$= -x^2 \quad \text{if } x < 0$$

$$f'(x) = 2x \quad \text{if } x \geq 0$$
$$= -2x \quad \text{if } x < 0$$

Q Nos 16 to 20 are very short answ
type questions

16. Show that the function $y = ax + 2$
is a solution of the differential equ

$$2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$$

$$y = ax + 2a^2$$

$$\frac{dy}{dx} = a$$

$$2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$$

$$2(a)^2 + x(a) - (ax + 2a^2) =$$

$$\cancel{2a^2} + \cancel{xa} - \cancel{ax} - \cancel{2a^2} = 0$$

$$0 = 0$$

$\Rightarrow$ $y = ax + 2a^2$ is a solution
of the given differential
equation.

17] Find adj A if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$A_{11} = 3 \quad A_{12} = -4 \quad A_{21} = 1 \quad A_{22} = 2$

$$\text{adj}\, A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$