SECTION A

Question number 1 to 4 carry 1 mark each.

1. Find the direction cosines of the line joining the points $P(4,3,-5)$ and $Q(-2,1,-8)$

Direction ratio $(-2-4, 1-3, -8+5)$

$(-6, -2, -3)$

Direction cosines

$$\left(\frac{-6}{\sqrt{(-6)^2+(-2)^2+(-3)^2}}, \frac{-2}{\sqrt{(-6)^2+(-2)^2+(-3)^2}}, \frac{-3}{\sqrt{(-6)^2+(-2)^2+(-3)^2}}\right)$$

$$\left(\frac{-6}{\sqrt{36+4+9}}, \frac{-2}{\sqrt{36+4+9}}, \frac{-3}{\sqrt{36+4+9}}\right)$$

$$\left(\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}\right)$$

or.

1) Find the value of p for which the following lines are perpendicular

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}, \quad \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\frac{x-1}{-3} = \frac{y-7}{p} = \frac{z-3}{2}, \quad \frac{x-1}{-3p} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Since lines are perpendicular

$$(-3)(-3p) + p(1) + 2(-5) = 0$$

$$9p + p - 10 = 0$$

$$10p = 10$$

$$p = 1$$

2. Find the integrating factor of the differential equation

$$x\frac{dy}{dx} - 2y = 2x^2$$

$$\frac{dy}{dx} - \frac{2}{x}y = \frac{2x^2}{x}$$

$$\frac{dy}{dx} - \frac{2}{x}y = 2x$$

$$\frac{dy}{dx} + Py = Q.$$

$$IF = e^{\int -\frac{2}{x}dx} = e^{-2\log x} = e^{\log x^{-2}} = x^{-2}$$

3. Q. A is a square matrix of order 3 with $|A| = 4$ Then what is the value of $|-2A|$

$$|-2A| = (-2)^3 |A| = -8(4) = -32$$

4. Q. $y = \sin^{-1} x + \cos^{-1} x$

$$y = \frac{\pi}{2} \quad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

$$\frac{dy}{dx} = 0.$$

Question Number 5 to 12 carry two marks each

5. Q. $A = \begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$

Find the matrix $B'A'$

$A' = \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ $B' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix}$

$B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$

$\begin{bmatrix} 12+63 & 4+56-4 & 28+35+8 \\ 9 & 8-4 & 5+8 \\ 6+36 & 2+32-12 & 14+20+24 \end{bmatrix}$

$\begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 48 \end{bmatrix}$

6. Find $\int_a^b \frac{\log x}{x} \, dx$.

Put $\log x = t$ $\quad x = a \quad \log a = t$

$\frac{1}{x} dx = dt \quad x = b \quad \log b = t$

$$\int_{\log a}^{\log b} t \, dt = \left[\frac{t^2}{2}\right]_{\log a}^{\log b}$$

$$= \left[\frac{\log b}{2}\right]^2 - \left[\frac{\log a}{2}\right]^2$$

7\. Form the differential equation representing the family of cu[rves] $y^2 = m(a^2 - x^2)$ by eliminati[ng] arbitrary constants m and a

$$y^2 = m(a^2 - x^2)$$

$$y^2 = ma^2 - mx^2$$

$$\cancel{2}y\frac{dy}{dx} = -\cancel{2}mx.$$

$$y\frac{dy}{dx} = -mx. \quad \text{I}$$

$$y\frac{d^2y}{dx^2} + 1.\frac{dy}{dx} = -m$$

Put this value in I

$$y\frac{dy}{dx} = x\left(y\frac{d^2y}{dx^2} + \frac{dy}{dx}\right.$$

8

$$\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$$

$$\int \frac{\sin(x-a+a-a)}{\sin(x+a)}\,dx$$

$$\int \frac{\sin((x+a)-2a)}{\sin(x+a)}\,dx$$

$$\int \frac{\sin(x+a)\cos 2a - \cos(x+a)\sin 2a}{\sin(x+a)}\,dx$$

$$\int \left(\frac{\cancel{\sin(x+a)}\cos 2a}{\cancel{\sin(x+a)}} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right)dx$$

$$\int \cos 2a\,dx - \int \cot(x+a)\,\cancel{dx}\,\sin 2a\,dx.$$

$$\cos 2a \int 1\,dx - \cancel{\sin 2a}\int \cot(x+a)\,dx$$

$$\cos 2a\,(x) - \sin 2a\ \log|\cancel{\sin}(x+a)| + c$$

$$x\cos 2a - \sin 2a\ \log|\sin(x+a)| + c$$

or

8. $\int (\log x)^2 \, dx$

$\int \underset{\text{II}}{1} \, \underset{\text{I}}{(\log x)^2} \, dx$.

$(\log x)^2 \cdot x - \int 2 \log x \times \frac{1}{\cancel{x}} \cancel{x} \, dx$.

$x(\log x)^2 - \int 2 \log x \, dx$

$x(\log x)^2 - 2 \int \underset{\text{II}}{1} \, \underset{\text{I}}{\log x} \, dx$

$x(\log x)^2 - 2 \left[(\log x) \cdot x - \int \frac{1}{x} \right.$

$x(\log x)^2 - 2 \left[x \log x - x \right] +$

$x(\log x)^2 - 2(x \log x - x) +$

9. Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\bar{a} \times \vec{b} = \begin{vmatrix} i & \bar{j} & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21)$$

$$0\hat{i} + 19\hat{j} + 19\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(0)^2 + (19)^2 + (19)^2} = 19\sqrt{2}.$$

$$= \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} = \frac{19\hat{j} + 19\hat{k}}{19\sqrt{2}}$$

$$\frac{19\hat{j}}{19\sqrt{2}} + \frac{19}{19\sqrt{2}}\hat{k}.$$

$$\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$

or

9. Show that the vectors
$\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$,
$\hat{i} - 3\hat{j} + 5\hat{k}$ are co-plane

Prove $\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$

$1(15 \cancel{+} 12) - (-2)(-10+4) + 3$

$3 + 2(-6) + 3(3) = 0$

$3 - 12 + 9 = 0$

$0 = 0$.

So the vectors are co-

9. Mother, Father and son line up at random for a family photograph. If A and B are two events given by 11
10

A is son on one end and B father in the middle Find $P(B|A)$

Sol. $S = \{MFS, MSF, FMS, FSM, SFM, SMF\}$

~~Son on one~~

A = Son on one end

$A = \{MFS, FMS, SFM, SMF\}$

B = Father in the middle

$B = \{MFS, SFM\}$

$A \cap B = \{MFS, SFM\}$

$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

$P(A) = \frac{4}{6} = \frac{2}{3}$

$P(B) = \frac{2}{6} = \frac{1}{3}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/3}{2/3} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{2} = \frac{1}{2}$$

12) Let X be a random variable which is 12
II assumes values x_1, x_2, x_3, x_4 such that

$$2P(X=x_1) = 3P(X=x_2) = P(X=x_3) = 5P(X=x_4)$$

Find the probability distribution of X

$$2P(X=x_1) = 3P(X=x_2) = P(X=x_3) = 5P(X=x_4) = k$$

$$2P(X=x_1) = k$$

$$P(X=x_1) = \frac{k}{2}$$

$$3P(X=x_2) = k$$

$$P(X=x_2) = \frac{k}{3}$$

$$P(X=x_3) = k$$

$$5P(X=x_4) = k$$

$$P(X=x_4) = \frac{k}{5}$$

$$\frac{k}{2} + \frac{k}{3} + k \pm \frac{k}{5} = 1$$

$$\frac{15k + 10k + 30k + 6k}{30} = 1$$

$$61k = 30$$

$$k = \frac{30}{61}$$

X	1	2	3	4
$P(X)$	$\frac{1}{\cancel{2}} \times \frac{\cancel{30}^{15}}{61}$ $15/61$	$\frac{\cancel{k}}{3} \frac{\cancel{30}^{10}}{61}$ $10/61$	$\frac{30}{61}$ $30/61$	$\frac{1}{5} \times \frac{\cancel{30}^{6}}{61}$ $6/61$

or

11. A coin is tossed 5 times Find the probability of getting

I at least 4 heads

II at most 4 heads.

The repeated tosses of a coin are Bernoulli trials

Let X denote the number of heads

$n = 5 \quad p = \frac{1}{2} \quad q = \frac{1}{2}$

$P(\text{at least } 4 \text{ head}) = P(X=4) + P(X=5)$

$= {}^5C_4 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_5 \left(\frac{1}{2}\right)^5$

$= 5\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 (6)$

$= \frac{1}{\cancel{32}_{16}} \times \cancel{6}3 = \frac{3}{16}$

$P(\text{at most } 4 \text{ head}) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

${}^5C_0\left(\frac{1}{2}\right)^5 + {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5$

$\left(\frac{1}{2}\right)^5 \left[1 + 5 + 10 + 10 + 5\right]$

$\frac{1}{32} \times 31 = \frac{31}{32}$

SECTION B

Questions number 5 to 12 carry 2 marks each

5. Q. $*$ is defined on the set R of all real
12 numbers by $*$: $a * b = \sqrt{a^2+b^2}$
Find the identity element if it exist in R w.r.t $*$

Let e be identity element for the binary operation in R

$a * e = a = e * a$

$a * e = a$

$\sqrt{a^2+e^2} = a$

Squaring both side we get.

$a^2 + e^2 = a^2$

$e^2 = 0$

$e = 0$

$e * a = a$

$\sqrt{e^2+a^2} = a$

Squaring both side

$e^2 + a^2 = a^2$

$e^2 = 0$

$e = 0.$

$\Rightarrow$ 0 is the identity element in R

Question number 13 to 23 carry 4 marks each 15

13. Find the value of x if

$$\tan\left[\sec^{-1}\left(\frac{1}{x}\right)\right] = \sin\left(\tan^{-1} 2\right) \quad \text{I} \quad x > 0$$

$$\sec^{-1}\frac{1}{x} = \theta$$

$$\sin\left(\tan^{-1} 2\right) = \alpha$$

$$\tan\alpha = 2.$$

$$\sec\theta = \frac{1}{x}.$$

$$\sin\alpha = \frac{2}{\sqrt{5}}$$

$$\tan\theta = \frac{\sqrt{1-x^2}}{x}.$$

$$\theta = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

I becomes

$$\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

$$\frac{1-x^2}{x^2} = \frac{4}{5}$$

$$5 - 5x^2 = 4x^2$$

$$9x^2 = 5$$

$$x^2 = \frac{5}{9}$$

$$x = \frac{\sqrt{5}}{3}$$

14. If $e^y(x+1) = 1$

Then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

$$e^y(x+1) = 1$$

$$e^y = \frac{1}{x+1}$$

$$\log e^y = \log\left(\frac{1}{x+1}\right)$$

$$y \log e = \log 1 - \log(x+1)$$

$$y = -\log(x+1)$$

$$\frac{dy}{dx} = -\frac{1}{x+1} \frac{d}{dx}(x+1)$$

$$\frac{dy}{dx} = -\frac{1}{x+1} = -(x+1)^{-1}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{(x+1)^2} \qquad \text{I}$$

$$\frac{d^2y}{dx^2} = +(x+1)^{-2} \qquad \text{II}$$

$$= \frac{1}{(x+1)^2}$$

From I and II

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

or.

14 Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

$$y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$$

$$\sin y = \frac{2^{x+1}}{1+4^x}$$

$$-1 \leq \sin y \leq 1$$

$$-1 \leq \frac{2^{x+1}}{1+4^x} \leq 1$$

Since $\frac{2^{x+1}}{1+4^x}$ is positive

So $\frac{2^{x+1}}{1+(2^2)^x} \leq 1$

$$2^x \cdot 2 \leq 1 + 2^{2x}$$

$$2 \leq \frac{1+2^{2x}}{2^x}$$

$$2 \leq \frac{1}{2^x} + 2^x.$$

which is true for all x.

Hence function is defined for all real numbers.

$$y = \sin^{-1} \frac{2^{x+1}}{1+4^x}. \qquad y = \sin^{-1} \frac{2^x.}{1+}$$

Put. $2^x = \tan\theta$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta)$$

$$y = 2\theta$$

$$= 2\tan^{-1}(2^x)$$

$$\frac{dy}{dx} = 2 \frac{1}{1+(2^x)^2} \frac{d}{dx} 2^x$$

$$= \frac{2}{1+4^x} 2^x \log 2$$

$$= \frac{2^{x+1}}{1+4^x} \log 2$$

15. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(a) strictly increasing (b) strictly decreasing

$$f(x) = 4x^3 - 6x^2 - 72x + 30$$

$$f'(x) = 12x^2 - 12x - 72$$

Critical points

$$f'(x) = 0$$

$$12x^2 - 12x - 72 = 0$$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \quad x = -2$$

$$(-\infty, -2) \quad (-2, 3) \quad (3, \infty)$$

In $(-\infty, -2)$ $f'(x) = (x-3)(x+2)$

$(-)(-) = +ive > 0$

So increasing in $(-\infty, -2)$

In $(-2, 3)$ $f'(x) = (x-3)(x+2)$

$(-)(+) = -ive < 0$

So decreasing in $(-2, 3)$

In $(3, \infty)$ $f'(x) = (x-3)(x+2)$

$= (+ive)(+ive) = +ive > 0$

So increasing in $(3, \infty)$

SECTION C

Questions number 13 to carry 4 marks each.

16\. Show that the relation R on the set Z of all integers given by

$R = \{(a,b) : 2 \text{ divides } (a-b)\}$

is an equivalence relation.

Sol. Reflexive :-

2 divides $(a-a)$ for all $a \in Z$

So it is reflexive

Symmetric :-

If $(a,b) \in R \Rightarrow 2$ divides $a-b$

$\Rightarrow 2$ divides $b-a$

Hence $(b,a) \in R$

So it is symmetric

Transitive

If $(a,b) \in R \Rightarrow 2$ divides $(a-b)$

$a-b = 2q$

If $(b,c) \in R \Rightarrow 2$ divides $(b-c)$

$b-c = 2q$.

$a-c = a+b+b-c$

$= (a-b)+(b-c)$

$= 2q + 2q$

$= 4q$

$a-c = 4q \Rightarrow a-c$ is even

so $a-c$ is divisible by 2

$(a,c) \in R \Rightarrow R$ is transitive